MATH 1241 **FINAL EXAM** **FALL 2012**

Part I, No Calculators Allowed

1. Evaluate the limit: $\lim_{x \to 2} \dfrac{x^2 - 4}{x^2 + 4x - 12}$.

   (a) 0
   (b) 0.25
   (c) 0.5
   (d) 1
   (e) Does not exist

2. Which of the following is the derivative of $g(x) = x^2 \cos(3x + 1)$?

   (a) $2x \sin(3x + 1)$
   (b) $-6x \sin(3x + 1)$
   (c) $2x \cos(3x + 1) - x^2 \sin(3x + 1)$
   (d) $2x \cos(3x + 1) + 3x^2 \sin(3x + 1)$
   (e) $2x \cos(3x + 1) - 3x^2 \sin(3x + 1)$

3. Find an equation for the line that is tangent to the graph of $f(x) = x^3 - 7x + 4$ at $x = 2$.

   (a) $y = -7x + 4$
   (b) $y = -7x + 12$
   (c) $y = -3x + 4$
   (d) $y = 5x - 12$
   (e) $y = 5x + 4$

4. For what values of x, if any, does the function $f(x) = 3x^4 - 32x^3 + 72x^2 + 10$ have a local minimum?

   (a) There is no local minimum
   (b) Only at $x = 0$
   (c) Only at $x = 2$
   (d) Only at $x = 6$
   (e) At $x = 0$ and at $x = 6$

5. Which of the following is the derivative of $f(x) = e^{3x^2+1}$?

(a) $6x\,e^{3x^2+1}$
(b) e^{3x^2+1}
(c) $6\,e^{3x^2}$
(d) e^{6x}
(e) $(3x^2 + 1)e^{3x^2}$

6. Which of the following is the derivative of $f(x) = \dfrac{x^3}{\sin(5x)}$?

(a) $\dfrac{3x^2}{\cos(5x)}$
(b) $\dfrac{3x^2}{5\cos(5x)}$
(c) $\dfrac{3x^2\sin(5x) - 5x^3\cos(5x)}{\sin^2(5x)}$
(d) $\dfrac{3x^2\sin(5x) + 5x^3\cos(x)}{\sin^2(5x)}$
(e) $\dfrac{5x^3\cos(5x) - 3x^2\sin(5x)}{\sin^2(5x)}$

7. A particle is traveling around the circle $x^2 + y^2 = 25$ where x and y are measured in inches. At the instant the particle is at the point $(3, 4)$, $dy/dt = 15\,in/sec$. Find dx/dt at this time.

(a) $-20\,in/sec$
(b) $-15\,in/sec$
(c) $-2.5\,in/sec$
(d) $15\,in/sec$
(e) $20\,in/sec$

8. Which of the following is the derivative of $f(x) = \tan(x)$?

(a) $-\cot(x)$
(b) $\cot(x)$
(c) $\sec(x)$
(d) $\sec^2(x)$
(e) $\sec(x)\tan(x)$

9. Which of the following is the slope of the line tangent to the curve $y^2 + 3x^2 + xy = 36$ at $(2, 4)$?

(a) -4
(b) -2
(c) -1.6
(d) $-4/3$
(e) $8/3$

10. The derivative of the function $f(x)$ is given by $f'(x) = 20x + 6x^{1/2}$. Find a formula for the function $f(x)$ given that $f(1) = 25$.

(a) $f(x) = 10x^2 + 4x^{3/2} + 11$
(b) $f(x) = 20x^2 + 6x^{3/2} - 1$
(c) $f(x) = 40x^2 + 6x^{3/2} - 21$
(d) $f(x) = 40x^2 + 9x^{3/2} - 24$
(e) $f(x) = 3x^{-1/2} + 22$

11. Evaluate the limit: $\lim_{x \to 2} \dfrac{21x + 2}{7x - 4}$.

(a) 0
(b) 3
(c) 4.4
(d) 8
(e) Does not exist

12. Which of the following is the derivative of $f(x) = \ln(8x + 3)$?

(a) $\dfrac{1}{8x + 3}$
(b) $\dfrac{8}{8x + 3}$
(c) $\dfrac{-64}{(8x + 3)^2}$
(d) $\dfrac{-8}{(8x + 3)^2}$
(e) $\dfrac{-1}{(8x + 3)^2}$

13. Evaluate the limit: $\lim_{x \to +\infty} \dfrac{9x + 3e^{-x}}{2x - 5e^{-x}}$.

(a) -4
(b) 0
(c) 6/7
(d) 4.5
(e) Does not exist

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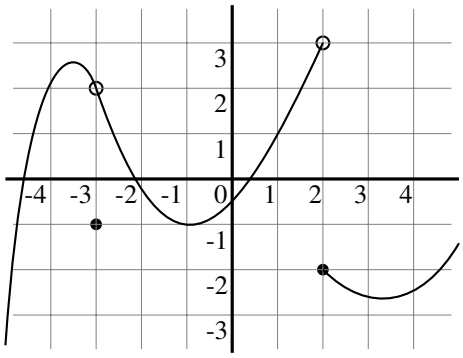
Part II, Calculators Allowed

1. Determine the values of A and B (if they exist) using the graph of $f(x)$.

$\lim_{x\to -3} f(x) = A$ $\lim_{x\to 2^-} f(x) = B$



(a) A does not exist, $B = -2$
(b) $A = -1$, $B = -2$
(c) $A = -1$, $B = 3$
(d) $A = 2$, $B = -2$
(e) $A = 2$, $B = 3$

2. A cube is measured to have edges of length 20 cm with a possible error no worse than ±0.03 cm. Use differentials to estimate the maximum error in calculating the volume.

(a) ± 60.0 cm^3
(b) ± 36.0 cm^3
(c) ± 24.0 cm^3
(d) ± 1.8 cm^3
(e) ± 0.6 cm^3

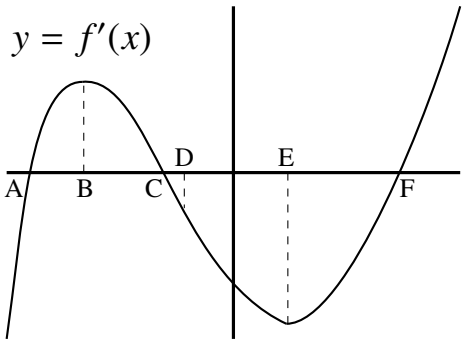
3. The function $f(x)$ has a derivative for each value of x and $g(x) = \sqrt{f(x)}$. Find $g'(2)$ given that $f(2) = 25$ and $f'(2) = 16$.

(a) $g'(2) = 3.2$
(b) $g'(2) = 1.6$
(c) $g'(2) = 0.2$
(d) $g'(2) = 0.1$
(e) $g'(2)$ does not exist

4. Use Newton's method to approximate where $f(x) = x^3 + x^2 + 2x + 3$ has a zero. Start with $x_1 = 1$ as the first approximation and calculate x_2 and x_3.

(a) $x_2 = 0$ and $x_3 = -1.5$
(b) $x_2 = 0$ and $x_3 = -2/3$
(c) $x_2 = 0$ and $x_3 = 1.5$
(d) $x_2 = 2$ and $x_3 = 55/18$
(e) $x_2 = 2$ and $x_3 = 56/19$

5. The graph at right is the graph of the **derivative** of the function $f(x)$ [so the graph of $y = f'(x)$]. Which of the following statements is true about the **function** $f(x)$.



(a) $f(x)$ is increasing when $A < x < C$ and $F < x < +\infty$ and concave up when $-\infty < x < B$ and $E < x < +\infty$

(b) $f(x)$ is increasing when $A < x < C$ and $F < x < +\infty$ and concave up when $D < x < +\infty$

(c) $f(x)$ is increasing when $-\infty < x < B$ and $E < x < +\infty$ and concave up when $D < x < +\infty$

(d) $f(x)$ is increasing when $-\infty < x < B$ and $E < x < +\infty$ and concave up when $A < x < B$ and $F < x < +\infty$

(e) $f(x)$ is increasing when $-\infty < x < B$ and $E < x < +\infty$ and concave up when $-\infty < x < D$

6. Which of the following limits represents the derivative of $f(x) = \cos(3x + 1)$?

(a) $\lim_{h\to 0} \dfrac{\cos(3x + h + 1) - \cos(3x + 1)}{h}$

(b) $\lim_{h\to 0} \dfrac{\cos(3x + 3h + 3) - \cos(3x + 1)}{h}$

(c) $\lim_{h\to 0} \dfrac{\cos(3x + 3h + 1) - \cos(3x + 1)}{h}$

(d) $\lim_{h\to 0} \dfrac{3\cos(x + h + 1/3) - 3\cos(x + 1/3)}{h}$

(e) $\lim_{h\to 0} \dfrac{\cos(3x + h + 1)}{h}$

7. The derivative of a function $g(x)$ is given by $g'(x) = -7(x + 3)^2(x - 1)(x - 5)$. Find the x-coordinates [only the x since you don't know what $g(x)$ is] for each local maximum and each local minimum of $g(x)$, if any.

(a) Local maxima at $x = 1$ and $x = 5$, local minimum at $x = -3$

(b) Local maximum at $x = 1$, local minimum at $x = 5$

(c) Local maxima at $x = -3$ and $x = 5$, local minimum at $x = 1$

(d) Local maximum at $x = 5$, local minimum $x = 1$

(e) Local maximum at $x = 1$, local minima at $x = -3$ and $x = 5$

8. A particle moves along the x-axis and its position at time t is given by $x(t) = 400t - t^3$ for $0 \leq t$ where t is measured in seconds and x in feet. What is the average velocity from $t = 5$ to $t = 10$?

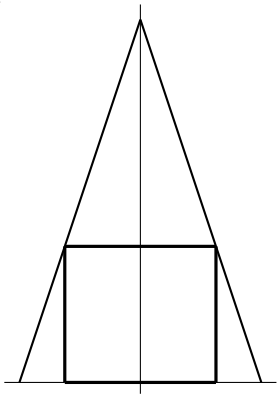
(a) 212.5 ft/sec
(b) 225 ft/sec
(c) 231.25 ft/sec
(d) 337.5 ft/sec
(e) 343.75 ft/sec

9. As in Problem #8, a particle moves along the x-axis and its position at time t is given by $x(t) = 400t - t^3$ for $0 \leq t$ where t is measured in seconds and x in feet. What is the instantaneous velocity at $t = 7.5$?

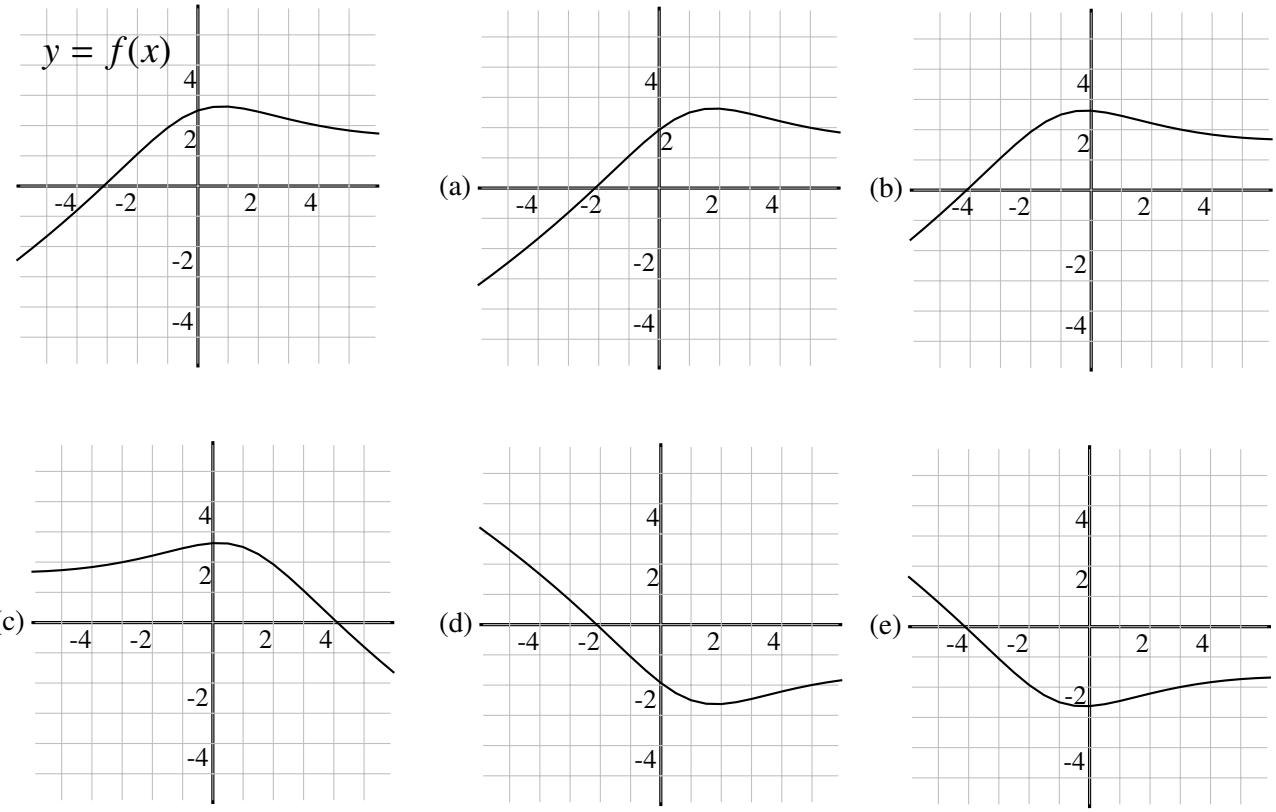
(a) 212.5 ft/sec
(b) 225 ft/sec
(c) 231.25 ft/sec
(d) 337.5 ft/sec
(e) 343.75 ft/sec

10. Find the maximum area of a rectangle that is inside the triangle formed by the x-axis and the lines $y = -3x + 12$ and $y = 3x + 12$ if the base of the rectangle is on the x-axis and the two upper vertices are on the lines $y = -3x + 12$ and $y = 3x + 12$ as in the illustration.

(a) 30
(b) 24
(c) 18
(d) 12
(e) 9



11. The first graph on the left below is the graph of $y = f(x)$. Which of the graphs labeled (a), (b), (c), (d) and (e) best represents the graph of $y = -f(x+1)$?



12. The second derivative of the function $f(x)$ is $f''(x) = 16x - x^3$. Find the x-coordinate of each inflection point of the function $f(x)$.

(a) Only inflection point is at $x = 0$
(b) Only inflection point is at $x = 4$
(c) There are two inflection points: at $x = -4$ and at $x = 4$
(d) There are three inflection points: at $x = -4$, at $x = 0$ and at $x = 4$
(e) There are no inflection points

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Part III, Calculators Allowed

1. Answer the questions below based on the following information about the function f. You must justify your answers.

(i) The function f is continuous and differentiable for all values of x.

(ii) $f(x) < 0$ for $x < 0$; $f(x) > 0$ for $0 < x$.

(iii) $f'(x) < 0$ for $-6 < x < -2$ and $5 < x$.

(iv) $f'(x) > 0$ for $x < -6$ and $-2 < x < 5$.

(v) $f''(x) < 0$ for $x < -4$ and $3 < x < 7$.

(vi) $f''(x) > 0$ for $-4 < x < 3$ and $7 < x$.

(a) On which intervals is the function decreasing?

(b) What is the x-coordinate of each local maximum (if any)?

(c) On which intervals is the function concave up?

(d) What is the x-coordinate of each inflection point (if any)?

2. Use the following table of values for (a), (b) and (c) below

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	4	-2	-3
2	4	1	7	5
3	1	2	13	-4
4	2	3	10	-8

(a) Find $b'(4)$ for $b(x) = \dfrac{g(x)}{f(x)}$.

(b) Find $h'(3)$ for $h(x) = g(f(x))$.

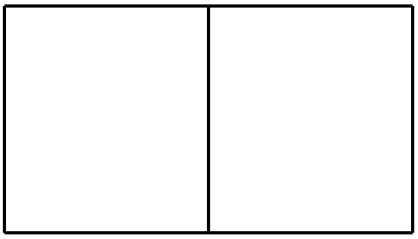
(c) Find $k'(2)$ for $k(x) = (f(x))^3$.

3. Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 - 150x^2 + 50{,}000$ on each interval.

(a) $-20 \le x \le 20$

(b) $-10 \le x \le 60$

4. A large rectangular area is to be fenced off as in the diagram below (a large rectangle divided into two smaller rectangles). The fence used to divide the space costs \$10 per foot and the fence used for the perimeter costs \$15 per foot. If the total budget for the project is \$60000, what are the dimensions which yield the largest area?



5. A spotlight at ground level is located 40 feet from a very tall building, directly in front of the door into the building. A 6 feet tall woman exits the building and walks directly towards the light. If she is walking at 5 feet per second, how fast is the length of her shadow on the building changing when she is 10 feet from the building?