Part I (MULTIPLE CHOICE, NO CALCULATORS).

1. Find $\int_1^2 4x^3\,dx$.

(a) 28

(b) 15

(c) 0

(d) 36

(e) 17

2. Find $\int 2\cos t\,dt$.

(a) $-2\sin t + C$

(b) $2\sin t + C$

(c) $2\cos t + C$

(d) $-2\cos t + C$

(e) $2\sin t\cos t + C$

3. Find $\int_0^1 (2x-1)^2\,dx$.

(a) 6

(b) 4/3

(c) 2/3

(d) 1

(e) 1/3

4. Find $\int_0^2 xe^{x^2-1}\,dx$.

(a) $2e^3$

(b) $2(e^3 - e^{-1})$

(c) $\frac{1}{2}(e^3 - e^{-1})$

(d) $2e^4$

(e) $e^3 - e^{-1}$

5. Find $\int x \ln x \, dx$.

(a) $x^2 \ln x - \frac{1}{2} \ln x + C$

(b) $x^2 \ln x + \frac{1}{2} \ln x + C$

(c) $\ln x + C$

(d) $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$

(e) $\frac{1}{2}x^2 \ln x + \frac{1}{4}x^2 + C$

6. The Maclaurin series (Taylor series centered at $a = 0$) for $f(x) = e^x$ is

(a) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

(b) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots$

(c) $1 + x + x^2 + x^3 + x^4 + \cdots$

(d) $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

(e) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

7. Which of the following definite integrals gives the length of the curve $y = \cos(x)$ for $0 \leq x \leq \pi$?

(a) $\int_0^\pi \sqrt{1 - \cos^2 x}\, dx$

(b) $\int_0^\pi \sqrt{1 + \cos^2 x}\, dx$

(c) $\int_0^\pi \sqrt{1 - \sin^2 x}\, dx$

(d) $\int_0^\pi \sqrt{1 + \sin^2 x}\, dx$

(e) $\int_0^\pi \sqrt{1 + \sin x}\, dx$

8. If $f(x) = \int_0^x e^{t^2+1}\, dt$, then $f'(x) =$

(a) e^{2x}

(b) e^x

(c) e^{x^2+1}

(d) $2xe^{x^2+1}$

(e) $2e^{2x}$

9. The average value of $f(x) = \dfrac{1}{2x+1}$ on $[1, 5]$ is

(a) $\dfrac{1}{8}(\ln 11 - \ln 3)$

(b) $\dfrac{1}{4}(\ln 11 - \ln 3)$

(c) $\dfrac{1}{2}(\ln 11 - \ln 3)$

(d) $\ln 11 - \ln 3$

(e) 3

10. Which of the following is equal to $\int \sin^2 x \cos x\, dx$?

(a) $\sin x \cos x + C$

(b) $-\dfrac{1}{3}\cos^3 x + C$

(c) $\dfrac{1}{3}\cos^3 x + C$

(d) $-\dfrac{1}{3}\sin^3 x + C$

(e) $\dfrac{1}{3}\sin^3 x + C$

11. In the Taylor series expansion of $f(x) = x^4$ centered at $a = 1$, what is the coefficient of $(x-1)^3$?

(a) 0

(b) 1

(c) 4

(d) 8

(e) 12

12. The following integral form appears in the Table of Integrals in your text:

$$\int \sqrt{a^2 - u^2}\, du = \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$$

Using this form and an appropriate substitution, we obtain $\int \sqrt{1 - 4x^2}\, dx =$

(a) $\frac{x}{2}\sqrt{1 - x^2} + \frac{1}{2}\sin^{-1} x + C$

(b) $x\sqrt{1 - 4x^2} + \frac{1}{2}\sin^{-1} 2x + C$

(c) $\frac{x}{2}\sqrt{1 - 4x^2} + \frac{1}{4}\sin^{-1} 2x + C$

(d) $\frac{x}{2}\sqrt{1 - 4x^2} + \frac{1}{2}\sin^{-1} 2x + C$

(e) $\frac{1}{4}\sqrt{1 - 4x^2} + \frac{1}{4}\sin^{-1} 2x + C$

13. Which of the following series converges?

(a) $\sum_{n=1}^{\infty} \frac{1}{2n}$

(b) $\sum_{n=1}^{\infty} \left(\frac{7}{4}\right)^n$

(c) $\sum_{n=1}^{\infty} \frac{n}{n^3 + 1}$

(d) $\sum_{n=1}^{\infty} (-1)^n$

(e) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

14. The geometric series $2 + \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \cdots$

(a) diverges.

(b) converges to 2.

(c) converges to $\frac{3}{2}$.

(d) converges to $\frac{8}{3}$.

(e) converges to $\frac{8}{5}$.

15. The region in the first quadrant bounded by the curves $y = x^2$, $y = 0$, and $x = 1$ is rotated about the x-axis. The volume of the resulting solid is

(a) $\frac{\pi}{5}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$

(d) 2

(e) $\frac{1}{3}$

16. The velocity, in meters per second, of a particle moving along a straight line is given by $v(t) = \sin t$. Find the distance traveled by the particle for $0 \leq t \leq \pi/2$.

(a) 1 m

(b) 2 m

(c) $\pi/2$ m

(d) $\pi/4$ m

(e) 0 m

Part II (MULTIPLE CHOICE, CALCULATORS ALLOWED).

1. $\int_{-1}^{1} \mid x \mid \, dx =$

(a) $1/2$

(b) $-1/2$

(c) 0

(d) 1

(e) -1

2. Let $a_n = \dfrac{2n+1}{4n+1000}$ for $n = 1, 2, 3, \ldots$. Which of the following statements is true?

(a) The sequence $\{a_n\}$ converges to 0.

(b) The sequence $\{a_n\}$ converges to 1.

(c) The sequence $\{a_n\}$ converges to 2.

(d) The sequence $\{a_n\}$ converges to $\dfrac{1}{2}$.

(e) The sequence $\{a_n\}$ diverges.

3. The improper integral $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\, dx$

(a) converges to 1.

(b) converges to 0.

(c) converges to 2.

(d) diverges to ∞.

(e) diverges to $-\infty$.

4. Consider the series $\sum_{n=1}^{\infty} \frac{2n+1}{n^2}$. Which of the following results in a successful determination of the convergence or divergence of this series?

(a) Comparison with the terms of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(b) Comparison with the terms of the series $\sum_{n=1}^{\infty} \frac{1}{n}$

(c) The alternating series test.

(d) The test for divergence ($\lim_{n\to\infty} \frac{2n+1}{n^2} \neq 0$).

(e) The ratio test.

5. Let $f(x) = x^2$. Find the Riemann sum for f on the interval $[0, 2]$, using 4 subintervals of equal width and taking the sample points to be the left endpoints. (Round your answer to two decimal places.)

(a) 1.67

(b) 3.50

(c) 1.75

(d) 0.88

(e) 2.75

6. Recall that Hooke's Law states that the force required to maintain a spring stretched x units beyond its natural length is given by kx, where k is the spring constant. Suppose that a force of 3 lb is required to hold a spring stretched 0.5 ft beyond its natural length. How much work is done in stretching it from its natural length to 1 ft beyond its natural length?

(a) 12 ft-lb

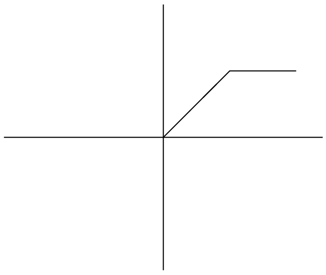
(b) 2 ft-lb

(c) 3 ft-lb

(d) 1 ft-lb

(e) 8 ft-lb

7. Consider the graph of the function f:



Find $\int_0^2 f(x)\,dx$.

(a) 3

(b) $\dfrac{5}{2}$

(c) 2

(d) $\dfrac{3}{2}$

(e) 1

8. What is the interval of convergence for the following power series: $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{3n}$?

(a) $(-\infty, \infty)$

(b) $(-3, 3)$

(c) $(-1, 1)$

(d) $[-3, -1)$

(e) $(-3, -1)$

9. Which of the following is the approximation obtained when the midpoint rule with $n = 3$ subintervals is used to approximate $\displaystyle\int_1^4 \frac{1}{x}\,dx$. (Round your answer to 4 decimal places.)

(a) 1.3524

(b) 1.3863

(c) 1.3015

(d) 1.4172

(e) 1.2537

10. What is the area of the region bounded above by the curve $y = 1$ and below by the curve $y = \dfrac{1}{x}$ for $1 \leq x \leq 2$? (Round the answer to one decimal place.)

(a) 0.9

(b) 0.7

(c) 0.5

(d) 0.3

(e) 1.0

11. For each real number x, let $f(x) = 1 + \dfrac{x}{2!} + \dfrac{x^2}{4!} + \cdots = \displaystyle\sum_{n=0}^{\infty} \frac{x^n}{(2n)!}$. Then $f'(0) =$

(a) 0

(b) 1/2

(c) 1

(d) 3/2

(e) 2

12. An in-ground circular swimming pool has a diameter of 20 ft and is 5 ft deep. If it is filled to a depth of 2 ft, find the work done in pumping all the water out at ground level. (Water weighs 62.5 lb/ft^3.)

(a) $50,000\pi$ ft-lb

(b) $6,250\pi$ ft-lb

(c) 100π ft-lb

(d) $1,600\pi$ ft-lb

(e) $100,000\pi$ ft-lb

13. Find the area of the finite region bounded by the curves $y = 4x - x^2$ and $y = x$, and round the answer to one decimal place.

(a) 5.3

(b) 5.7

(c) 4.5

(d) 18.0

(e) 0

14. Which of the following is true about the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n^2}$?

(a) The series diverges to ∞.

(b) The series is absolutely convergent.

(c) The series is conditionally convergent but not absolutely convergent.

(d) The series diverges to $-\infty$.

(e) The series is a geometric series.

Part III (FREE RESPONSE, CALCULATORS ALLOWED).

Note: Even though calculators are allowed, you must show your work in order to receive credit.

1. (a) (5 points) Find $\int xe^{3x}\,dx$.

(b) (5 points) Find $\int \frac{5x+1}{(x-1)(x+2)}\,dx$.

2. Consider the region in the first quadrant bounded by the curves $y = \sqrt{x}$, $y = 1$, and the y-axis.

(a) (5 points) Find the volume of the solid formed when this region is rotated about the y-axis.

(b) (5 points) Find the volume of the solid formed when this region is rotated about the line $x = 1$.

3. Consider the region in the first quadrant bounded by the curve $y = 1/x^3$ and the lines, $x = 1$, $x = 10$, and the x-axis.

(a) (5 points) Find the area of this region.

(b) (5 points) Find the x-coordinate of the centroid of this region.

4. Recall the power series expansion $\dfrac{1}{1+t} = 1 - t + t^2 - t^3 + \cdots = \sum_{n=0}^{\infty}(-1)^n t^n$, valid on the interval $(-1, 1)$.

(a) (4 points) Use the given expansion to find a power series expansion for $\dfrac{1}{1+t^2}$.

(b) (3 points) Use the expansion you derived in part (a) to find a power series expansion for $\tan^{-1} x = \displaystyle\int_0^x \frac{dt}{1+t^2}$.

(c) (3 points) Use the expansion in part (b) to approximate $\tan^{-1} 0.1$ to within 10^{-7}.

5. (a) (7 points) Use Simpson's rule with $n = 4$ subintervals to approximate $\int_1^2 \ln x \, dx$.

(b) (3 points) The error estimate when Simpson's rule is used to approximate $\int_a^b f(x)\, dx$ is given by

$$|E_n| \leq K \frac{(b-a)^5}{180n^4},$$

where n is the (even) number of subintervals and K is an upper bound for $|f^{(4)}(x)|$ on $[a, b]$. (Recall that $f^{(4)}$ is the fourth derivative of f.) Use this estimate to find an upper bound on the error in your approximation in part (a).