MATH 1241 Common Final Exam
Fall 2010

Please print the following information:

Name:	____________________	Instructor:	____________________
Student ID:	____________________	Section/Time:	____________________

The MATH 1241 Final Exam consists of three parts. You have three hours for the entire test. But you have only one hour to complete the first part. You **must turn in Part I at 9:00 a.m.** You can start working on Part II or Part III as soon as you finish working with Part I, however you are not allowed to use calculators until 9:00 a.m. These pages contain Part I which consists of 15 multiple choice questions. A special answer sheet is provided so that your answers can be machine graded.

- You must use a pencil with a soft black lead (# 2 or HB) to enter your answers on the answer sheet.
- For each question choose the response which *best* fits the question.
- If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.
- There is no penalty for guessing. However if you mark more than one answer to a question, that question will be scored as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.
- Make sure that your *name* appears on the answer sheet and that you fill in the circles corresponding to your name.
- At the end of the examination you MUST hand in this booklet, your answer sheet and all scratch paper.

Part I (CALCULATORS ARE NOT ALLOWED).

1. The derivative of

$$f(x) = x^2 + x - 1$$

(a) is equal to $2x$
(b) is equal to $2x - 1$
(c) is equal to $2x + 1$
(d) is equal to $x + 1$
(e) is equal to $x^2 + x$

2. The derivative of

$$f(x) = \frac{1}{x^9}$$

(a) is equal to $-9/x^9$
(b) is equal to $1/x^{10}$
(c) is equal to $9/x^{10}$
(d) is equal to $-9/x^{10}$
(e) is equal to $1/(9x^8)$

3. The limit

$$\lim_{x\to\infty} \frac{x^2}{x^4 + 1}$$

(a) is equal to 2
(b) is equal to 5
(c) is equal to -1
(d) is equal to ∞
(e) is equal to 0

4. Find the value of the parameter b for which the function

$$f(x) = \begin{cases} -x, & \text{if } x < 1, \\ 2x + b, & \text{if } x \geq 1. \end{cases}$$

is continuous on the interval $(-\infty, \infty)$

(a) $b = -3$
(b) $b = 3$
(c) $b = 1$
(d) $b = 0$
(e) $b = -1$

5. The equation of the tangent line to the graph of the function

$$f(x) = e^{2x} - 1$$

at the point with coordinates $x = 0$, $y = 0$

(a) is $y = 2x + 1$
(b) is $y = -2x$
(c) is $y = 7x$
(d) is $y = 9x$
(e) is $y = 2x$

6. The derivative of the function $f(x) = \ln(1 + x^8)$ equals

(a)
$$\frac{8x^7}{1 + x^8}$$

(b)
$$\frac{1}{1 + 8x^7}$$

(c)
$$\frac{x^7}{1 + x^8}$$

(d)
$$\frac{1}{1 + x^8}$$

(e)
$$\ln(8x^7)$$

7. According to L'Hospital's rule, the limit

$$\lim_{x \to 0} \frac{\ln(1+4x)}{2x}$$

(a) is equal to 3
(b) is equal to 1
(c) is equal to 5
(d) is equal to ∞
(e) is equal to 2

8. The derivative of the function

$$f(x) = e^x \cos(x)$$

equals

(a) $e^x(\cos(x) - \sin(x))$
(b) $e^x \sin(x)$
(c) $e^x \cos(x)$
(d) $e^x(\sin(x) + \cos(x))$
(e) $2\sin(x) + 2\cos(x)$

9. The derivative of the function

$$f(x) = \sqrt{x^2 + 2x + 5}$$

equals

(a) $f'(x) = \sqrt{2x+2}$
(b) $f'(x) = \frac{x+1}{\sqrt{x^2+2x+5}}$
(c) $f'(x) = 2x + 2$
(d) $f'(x) = \frac{2x+2}{\sqrt{x^2+2x+5}}$
(e) $f'(x) = x^2$

10. The derivative of the function

$$f(x) = \sin(e^{x^2})$$

equals

(a) $f'(x) = 2x \sin(e^{x^2})$
(b) $f'(x) = 2x \cos(e^{x^2})$
(c) $f'(x) = e^{x^2} \cos(e^{x^2})$
(d) $f'(x) = xe^{x^2} \cos(e^{x^2})$
(e) $f'(x) = 2xe^{x^2} \cos(e^{x^2})$

11. Let$f(x) = \arctan(x^5) = \tan^{-1}(x^5)$. Then $f'(x) =$

(a)

$$\frac{5x^4}{x^2+1}$$

(b)

$$\frac{1}{x^{10}+1}$$

(c)

$$\frac{5x^4}{x^{10}+1}$$

(d)

$$\frac{5x^4}{x^{10}+2}$$

(e)

$$\frac{1}{x^2+1}$$

12. The derivative of the function $f(x) = \frac{e^x}{x^2+1}$ equals

(a) $\frac{(x^2+2)e^x}{(x^2+1)^2}$
(b) $\frac{(x-1)e^x}{(x^2+1)^2}$
(c) $\frac{x^2e^x}{(x^2+1)^2}$
(d) $\frac{(x^2+2x+1)e^x}{(x^2+1)^2}$
(e) $\frac{(x^2-2x+1)e^x}{(x^2+1)^2}$

13. The most general anti-derivative of the function

$$f(x) = 3x^2 + 4x + 3$$

(a) is $x^3 + 2x^2 + 3x + C$
(b) is $x^3 + 2x^2 + 2x + C$
(c) is $5x^3 + 2x^2 + 3x + C$
(d) is $x^3 + 2x + 3x + C$
(e) is $3x^3 + x^2 + 3x + C$

14. If $f(x) = \sin(5x)$, then the second derivative $f''(x)$ of f equals

(a) $5\cos(5x)$
(b) $-25\sin(5x)$
(c) $25\cos(5x)$
(d) $\cos(5x)$
(e) $-\sin(5x)$

15. The maximum **value** of the function

$$f(x) = \frac{6x}{x^2 + 9}$$

equals

(a) 7
(b) 5
(c) -1
(d) 1
(e) 2

Part II (CALCULATORS ARE ALLOWED).

Please print the following information:

Name:	____________________	Instructor:	____________________
Student ID:	____________________	Section/Time:	____________________

The MATH 1241 Final Exam consists of three parts. These pages contain Part II which consists of 12 multiple choice questions. A special answer sheet is provided so that your answers can be machine graded.

- You must use a pencil with a soft black lead (# 2 or HB) to enter your answers on the answer sheet.
- For each question choose the response which *best* fits the question.
- If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.
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Part II (CALCULATORS ARE ALLOWED).

1. Let $f(x)$ be the piecewise continuous function defined by

$$f(x) = \begin{cases} x, & \text{if} \quad x < 1; \\ 2 - x, & \text{if} \quad 1 \leq x \leq 5; \\ 0, & \text{if} \quad x > 5. \end{cases}$$

Choose the correct statement:

(a) $\lim_{x\to 5^-} f(x) = 0$
(b) $\lim_{x\to 5^+} f(x) = f(5)$
(c) $f(x)$ is discontinuous at $x = 0$
(d) $f(x)$ is continuous at $x = 5$
(e) $f(x)$ is continuous at $x = 1$

2. Suppose that the derivative of f equals

$$f'(x) = (x + 1)(3 - x)$$

Choose the correct statement:

(a) f is increasing on the interval $(3, \infty)$
(b) f is increasing on the interval $(-\infty, -1)$
(c) f is increasing on the interval $(-1, 3)$
(d) f has a local minimum at $x = 3$
(e) f has a local maximum at $x = -1$

3. Suppose that the derivative of f equals

$$f'(x) = x(10 - x)$$

Choose the correct statement:

(a) f is concave upward on the interval $(5, \infty)$
(b) f is concave upward on the interval $(-\infty, 5)$
(c) f has a maximum at the point $x = 5$
(d) f has an inflection point at $x = 0$
(e) f has an inflection point at $x = 10$

4. Let $f(x)$ be a continuous function on the closed interval $[0,5]$, such that $f(0) = 2$, $f(5) = 4$. Use the Intermediate Value Theorem to find the integer number A, for which the equation $f(x) = A$ always has a solution on the interval $(0,5)$, no matter what function f with the properties described above we have.

(a) $A = 2$
(b) $A = 1$
(c) $A = 5$
(d) $A = 4$
(e) $A = 3$

5. Let $f(x) = 2x^2$ and $g(x) = 5x^7$. Then $f(g(x))$ equals

(a) $25x^9$
(b) $10x^{14}$
(c) $50x^9$
(d) $10x^9$
(e) $50x^{14}$

6. The following table gives values of functions $f(x)$ and $g(x)$ as well as their first derivatives for different values of x.

x	1	2	3	4	5	6	7	8	9
$f(x)$	2	4	6	8	10	12	14	16	18
$g(x)$	1	2	4	7	9	5	3	2	1
$f'(x)$	1	2	3	1	2	1	3	1	2
$g'(x)$	1	2	2	3	0	-1	-2	-3	-1

According to this table, the **derivative** of $f \circ g$ at the point $x = 3$ equals
(a) 2
(b) 6
(c) 1
(d) -3
(e) -1

7. Which of the following limits represents the value of the derivative of $f(x)$ at the point 2?

(a)

$$f'(2) = \lim_{h\to 0} \tfrac{f(h)-f(0)}{h}$$

(b)

$$f'(2) = \lim_{h\to 0} \tfrac{f(2+2h)-f(2)}{h}$$

(c)

$$f'(2) = \lim_{h\to 0} \tfrac{f(2+h)-f(2)}{h}$$

(d)

$$f'(2) = \lim_{h\to 0} \tfrac{f(2-h)-f(2)}{h}$$

(e)

$$f'(2) = \lim_{h\to 2} \tfrac{f(2+h)-f(2)}{h}$$

8. Consider a particle moving along a straight horizontal line. Let $x(t) = t^2 + t + 1$ be the position of the particle at the time t (i.e. $x(t)$ is the signed distance from the origin). Find its acceleration $a(t)$ as a function of t. (Here x, t and a are measured in cm, sec and cm/sec^2 correspondingly)

(a) $a(t) = 2t$
(b) $a(t) = 0$
(c) $a(t) = 2$
(d) $a(t) = t^2 + t + 1$
(e) $a(t) = 2t + 1$

9. The equation $7y^3x^5 + 3x^2y^9 = 10$ defines implicitly a function $y = y(x)$, such that $y(1) = 1$. Find $y'(1)$ using implicit differentiation.

(a) -1

(b) $\frac{70}{48}$

(c) 2

(d) 5

(e) $-\frac{41}{48}$

10. Suppose that we apply Newton's method to find the root of the equation

$$x^2 - 2x - 3 = 0.$$

Find the second approximation x_2 to the root, if it is known that the first approximation $x_1 = 5$

(a) 3

(b) $\frac{7}{2}$

(c) 5

(d) 4

(e) $\frac{1}{2}$

11. Let f, g and h be three functions having the property

$$g(x) + 3 \leq f(x) \leq h(x) - 2 \quad \text{for all} \quad x$$

and let

$$\lim_{x \to a} g(x) = 2, \qquad \lim_{x \to a} h(x) = 7.$$

Then $\lim_{x \to a} f(x)$ equals

(a) 0
(b) 7
(c) 5
(d) 3
(e) 2

12. The following table gives values of functions $f(x)$ and $g(x)$ as well as their first derivatives for different values of x.

x	1	2	3	4	5	6	7
$f(x)$	2	4	6	8	10	12	14
$g(x)$	1	2	4	7	9	5	3
$f'(x)$	1	2	3	1	2	3	1
$g'(x)$	1	2	2	3	0	-1	-2

According to this table, the derivative of the product fg at the point $x = 5$ equals

(a) 2
(b) 6
(c) 18
(d) -3
(e) 20

Part III (CALCULATORS ARE ALLOWED).

Please print the following information:

Name:	____________________	Instructor:	____________________
Student ID:	____________________	Section/Time:	____________________

The MATH 1241 Final Exam consists of three parts. These pages contain Part III consisting of free response questions. You must show your work in order to receive a credit. Empty or loose paper will not be graded.

- If you are basing your answer on a graph obtained by a calculator, sketch the graph. Be sure to put a scale on the x and y axes.
- Make sure that your name appears on each page.
- **At the end of the examination you MUST hand in all materials including this booklet, your answer sheet and all scratch paper.**

Problem	1	2	3	4	5
Grade					

FREE RESPONSE SCORE:

Part III (FREE RESPONSE QUESTIONS, CALCULATORS ARE ALLOWED).

You must show your work in order to receive a credit.

1. [Related Rates] Two boats move on the same (flat) lake. At noon boat B is 10 miles east of boat A. For the rest of the day, both boats sail at 10 mph, however boat A sails north and boat B sails east. We want to find out how fast the distance between the boats is changing at 3 pm. Answer the following questions.

(a) Let x be the distance that boat B has traveled east since noon and let y be the distance A has traveled north since noon. Express the distance d between the boats in terms of x and y (Draw a diagram and remember that B started 10 miles east of A at noon.)

(b) Use the Chain Rule to differentiate d with respect to t. Your answer should involve $\frac{dx}{dt}$ and $\frac{dy}{dt}$

(c) Find the values of x and y at 3 pm.

(d) Use the information in (b) and (c) to calculate the rate of change of the distance between the boats at 3 pm

2. [Sketch the Graph] Sketch the graph of a function f having the following properties:

(a) Its first derivative f' is positive on intervals $(-\infty, 0)$ and $(1, 3)$

(b) Its second derivative f'' is positive on the intervals $(-\infty, 0)$, $(0, 2)$ and $(4, \infty)$

(c) Its first derivative f' is negative on the intervals $(0, 1)$ and $(3, \infty)$

(d) Its second derivative f'' is negative on $(2, 4)$

(e)

$$\lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x) = 0$$

(f)

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) = \infty$$

(g) The values of f at the points 1, 2 and 3 are $f(1) = -1$, $f(2) = 0$ and $f(3) = 2$

(h) The domain of f is the union $(-\infty, 0) \cup (0, \infty)$. The first and the second derivatives are continuous on the domain.

3. [Study the Behavior of a Function] Let $f(x) = xe^{-2x}$. Answer the following questions (Use calculus and algebra, NOT just your graphing calculator to answer these questions)

(a) Calculate the derivative $f'(x)$ and find all critical points of f

(b) Where is the function $f(x)$ increasing and where is it decreasing?

(c) Find the point where f has a maximum

(d) Find the x-coordinate of the inflection point of the graph of f

4. [**Optimization Problem**] Consider a right circular cylinder, having the property that the area of its surface equals 6π square feet. Find the largest possible volume of this cylinder.

Let the radius of the cross section of the cylinder be equal to r and let the height of the cylinder be equal to h. Then the surface area of the cylinder equals

$$S = 2\pi r^2 + 2\pi r h.$$

(a) Use the condition that the surface area of the cylinder equals 6π to obtain the volume V of the cylinder as a function of only one variable r. (Remember that $V = \pi r^2 h$)

(b) Find the value of r that maximizes the volume V.

(c) What is the maximum volume V? (Express the result in multiples of π.)

5. [**Inflection points and maximal points**] Find the value of the parameters a and b for which the function

$$f(x) = x^3 - ax^2 - bx + 7$$

has a maximum at $x = -1$ and an inflection point with the x-coordinate equal to 1.

(a) Find the first derivative of f

(b) Find the second derivative of f

(c) Use the condition that $x = 1$ is the x-coordinate of the inflection point to find a

(d) Use the condition that f has a maximum at $x = -1$ to find b