PART I (Calculators Not Allowed)

1. Evaluate the integral $\int_{-1}^{2} (4x - 5)\, dx$.

   (a) -9 (b) -5 (c) 1 (d) 5 (e) 9

2. Evaluate the integral $\int_{1}^{4} |2x - 6|\, dx$.

   (a) -3 (b) 3 (c) 5 (d) 9 (e) 21

3. Evaluate the integral $\int \dfrac{1 + \sin^2 x}{\sin^2 x}\, dx$.

   (a) $1 + \cot x + C$ (b) $x - \cot x + C$ (c) $x + \cot x + C$ (d) $x + \csc x + C$ (e) $x - \csc x + C$

4. Evaluate the integral $\int_{1}^{2} (4x + 1)\ln x\, dx$.

   (a) $7\ln 2 - 1$ (b) $7\ln 2 - 3$ (c) $10\ln 2 - 1$ (d) $10\ln 2 - 3$ (e) $10\ln 2 - 4$

5. Evaluate the integral $\int_{0}^{2} x e^{x^2 - 1}\, dx$.

   (a) $2\,e^3$ (b) $2(e^3 - e^{-1})$ (c) 1 (d) $\frac{1}{2}(e^3 - e^{-1})$ (e) 2

6. Evaluate the improper integral $\displaystyle\int_1^\infty \frac{16x}{(x^2+1)^3}\,dx$.

(a) $\dfrac{1}{3}$ (b) $\dfrac{1}{2}$ (c) $\ 1$ (d) $\ 2$ (e) $\dfrac{8}{3}$

7. Evaluate the integral $\displaystyle\int \sqrt{4 - \frac{x}{3}}\,dx$.

(a) $C - 2\left(4 - \dfrac{x}{3}\right)^{3/2}$

(b) $C - \left(4 - \dfrac{x}{3}\right)^{3/2}$

(c) $3\left(4 - \dfrac{x}{3}\right)^{3/2} + C$

(d) $-6\left(4 - \dfrac{x}{3}\right)^{3/2} + C$

(e) $C - \dfrac{1}{2}\left(4 - \dfrac{x}{3}\right)^{3/2}$

8. Evaluate the integral $\displaystyle\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$.

(a) $\dfrac{\sin x^{3/2}}{x^{3/2}} + C$ (b) $\cos\dfrac{2}{\sqrt{x}} + C$ (c) $\sin\dfrac{2}{\sqrt{x}} + C$ (d) $2\cos\sqrt{x} + C$ (e) $2\sin\sqrt{x} + C$

9. Evaluate the integral $\displaystyle\int \frac{4\,dx}{(1-2x)^3}$

(a) $\dfrac{-1}{(1-2x)^2} + C$

(b) $\dfrac{1}{(1-2x)^2} + C$

(c) $\dfrac{1}{(1-2x)^4} + C$

(d) $\dfrac{-2}{(1-2x)^4} + C$

(e) $\dfrac{2}{(1-2x)^2} + C$

10. The series $\sum_{n=1}^{\infty}(-1)^n \dfrac{10^n\, n^2}{n!}$

    (a) converges absolutely

    (b) converges conditionally

    (c) diverges by the comparison test

    (d) diverges by the ratio test

    (e) diverges by the root test

11. What is the coefficient of $(x+3)^2$ in the Taylor expansion of $f(x) = (x+1)^3$ about $a = -3$?

    (a) -1 (b) -2 (c) -3 (d) -4 (e) -6

12. Determine whether the series $\sum_{n=0}^{\infty} \dfrac{4^{n+1}}{5^n}$ is convergent or divergent. If convergent, find the sum.
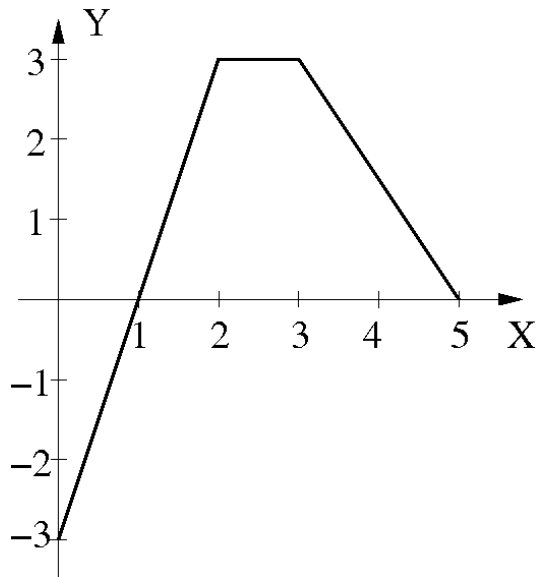
    (a) $\dfrac{15}{4}$ (b) $\dfrac{21}{4}$ (c) $\dfrac{27}{5}$ (d) 20   (e) diverges

13. The coefficient of x^5 in the power series for the function $f(x) = \dfrac{x}{1 + 3x^2}$ equals

    (a) -3 (b) 3   (c) -9 (d) 9   (e) 0

PART II (Calculators Allowed)

1. The graph of $y = f(x)$ is shown below. Estimate the value of the integral $\int_0^5 f(x)\,dx$.

     (a) 6   (b) 7   (c) 8   (d) 9   (e) 10

2. If $\int_1^7 f(x)\,dx = 4$, $\int_1^3 f(x)\,dx = 3$, and $\int_5^7 f(x)\,dx = 2$, find $\int_3^5 f(x)\,dx$.

     (a) 9   (b) 5   (c) 3   (d) 1   (e) -1

3. Express the limit $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i^2}{n^3}$ as a definite integral and evaluate it.

     (a) $\dfrac{1}{5}$ (b) $\dfrac{1}{4}$ (c) $\dfrac{1}{3}$ (d) $\dfrac{1}{2}$ (e) 1

4. Find the average value of the function $f(x) = 4x^3 + 2$ on the interval $[-2, 1]$.

     (a) -9 (b) -5 (c) -3 (d) 5   (e) $\dfrac{23}{3}$

5. Which of the following sequences converges?

I. $\dfrac{2n+3}{n-2}$ II. $\dfrac{\ln n^3}{n^2}$ III. $(-1)^n \dfrac{4n-3}{3n+2}$

(a) I only   (b) II only   (c) III only   (d) I and II only   (e) II and III only

6. Which of the following definite integrals gives the length of the curve $y = \cos(x)$ for $0 \leqslant x \leqslant \pi$.

(a) $\int_0^\pi \sqrt{1 + \cos(x)}\, dx$

(b) $\int_0^\pi \sqrt{1 + \cos^2(x)}\, dx$

(c) $\int_0^\pi \sqrt{1 - \cos^2(x)}\, dx$

(d) $\int_0^\pi \sqrt{1 + \sin(x)}\, dx$

(e) $\int_0^\pi \sqrt{1 + \sin^2(x)}\, dx$

7. Which of the following series converge?

I. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3^n}{n2^n}$ II. $\displaystyle\sum_{n=1}^{\infty} \frac{10^n}{n!}$ III. $\displaystyle\sum_{n=1}^{\infty} \frac{n^8}{4^n}$

(a) I only   (b) II only   (c) III only   (d) I and II only   (e) II and III only

8. Find the derivative of the function $f(x) = \displaystyle\int_0^{\sqrt{x}} \frac{2t}{t^4 + 1}\, dt$.

(a) $\dfrac{2\sqrt{x}}{x^2 + 1}$ (b) $\dfrac{1}{x^2 + 1}$ (c) $\dfrac{1}{x^4 + 1}$ (d) $\dfrac{2 - 6x^2}{(x^2 + 1)^2}$ (e) $\dfrac{2\sqrt{x}}{x^4 + 1}$

9. What work is done by the force $F(x) = 3x^2 - 4x + 7$ acting on a particle moving along the x-axis from $x = -1$ to $x = 2$?

(a)   10   (b)   18   (c)   20   (d)   22   (e)   24

10. Suppose that $f'(x) = \dfrac{4}{x^2} - 3x^2$ and $f(1) = 6$. Find a formula for $f(x)$ and then use it to evaluate $f(2)$.

(a) $f(2) = -7$ (b) $f(2) = -3$ (c) $f(2) = -1$ (d) $f(2) = 1$ (e) $f(2) = 3$

11. Find the x-coordinate of the centroid for the region bounded by $x = 0$, $y = 0$, and the line $y = -3x + 9$.

(a) $\dfrac{22}{27}$ (b) $\dfrac{8}{9}$ (c)   1   (d) $\dfrac{11}{9}$ (e) $\dfrac{29}{27}$

12. It is known that $\displaystyle\int \dfrac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\dfrac{u}{a} + C$. Use this formula to evaluate $\displaystyle\int \dfrac{dx}{\sqrt{9 - 4x^2}}$.
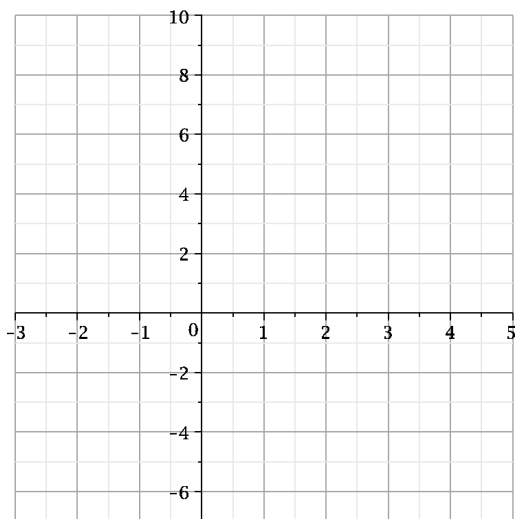
(a) $\dfrac{1}{2}\sin^{-1}\dfrac{2x}{3} + C$

(b) $\dfrac{1}{3}\sin^{-1}\dfrac{2x}{3} + C$

(c) $\dfrac{1}{3}\sin^{-1}\dfrac{4x}{3} + C$

(d) $\dfrac{1}{2}\sin^{-1}\dfrac{4x}{3} + C$

(e) $\sin^{-1}\dfrac{2x}{3} + C$

PART III (Calculators Allowed)

1. Find the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{n+1}$. Be sure to check the endpoints of the interval for convergence or divergence.

Answer:

2. Sketch the region enclosed by the curves $y = x^2 - 7$ and $y = 2x + 1$. Then find the area A of this region using an appropriate integral.

Answer: $A =$

3. (a) Find the partial fraction expansion of $R = \dfrac{2x - 8}{x^2 + 4x}$. Show your work.

Answer: $R =$

(b) Use the answer in part (a) to evaluate $I = \displaystyle\int \dfrac{2x - 8}{x^2 + 4x}\, dx$. Show your work.

Answer: $I =$

4. (a) Use the trapezoidal rule with $n = 3$ to estimate the value of $I = \int_1^4 \dfrac{dx}{2x - 1}$. Show your work.
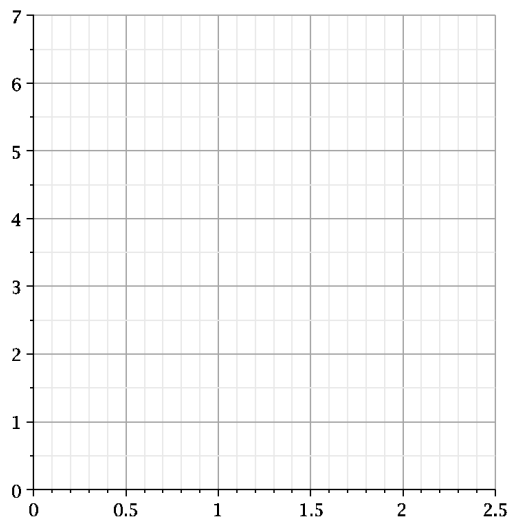
Answer: $\boxed{I \approx }$

(b) The error formula for estimating the value of $\int_a^b f(x)\,dx$ using the trapezoidal rule is

$|E_T| \leqslant \dfrac{K(b-a)^3}{12n^2}$, where $K \geqslant |f''(x)|$ for all $a \leqslant x \leqslant b$. Use this formula to estimate the value of $|E_T|$ in part (a) using the smallest possible value of K. Be sure to show how you find K.

Answer: $\boxed{K = }$ **Answer:** $\boxed{|E_T| \leqslant }$

5. Find the volume V of the solid obtained by rotating the region bounded by the curves $x = 0$, $y = 4$, and $y = x^2$ about the x-axis. Sketch a graph of the region.

Answer: $V =$

ANSWER KEY FOR PART I

1. (a)

2. (c)

3. (b)

4. (e)

5. (d)

6. (c)

7. (a)

8. (e)

9. (b)

10. (a)

11. (e)

12. (d)

13. (d)

ANSWER KEY FOR PART II

1. (a)

2. (e)

3. (c)

4. (c)

5. (d)

6. (e)

7. (e)

8. (b)

9. (e)

10. (d)

11. (c)

12. (a)

ANSWER KEY FOR PART III

1. $-\dfrac{1}{2} < x \leqslant \dfrac{1}{2}$

2. $A = 36$

3. (a) $R = \dfrac{2x + 2}{x^2 + 4} - \dfrac{2}{x}$ (b) $I = \ln(x^2 + 4) - 2\ln x + \arctan\dfrac{x}{2} + C$

4. (a) $I = \dfrac{116}{105} \approx 1.10476$ (b) $K = 8,\ |E_T| \leqslant 2$

5. $V = \dfrac{128\pi}{5} \approx 80.42477$