Part I (MULTIPLE CHOICE, NO CALCULATORS).

1. Find $\int_1^2 x^2\,dx$.

(a) $1/3$

(b) 2

(c) $7/3$

(d) $2/3$

(e) 3

2. $\int 3e^t\,dt = ?$

(a) $\dfrac{3}{t+1}e^{t+1} + C$

(b) $3e^t + C$

(c) $e^t + C$

(d) $3te^{t-1} + C$

(e) $3\ln|t| + C$

3. $\displaystyle\int_1^4 \frac{1}{u}\,du = ?$

(a) $e^4 - e^1$

(b) $-3/4$

(c) $\ln(3)$

(d) $\ln(4)$

(e) 3

4. Find the area of the region bounded above by the graph of $y = xe^{x^2}$ and below by the x-axis, $0 \le x \le 1$.

(a) $\frac{1}{2}e$

(b) $\frac{1}{2}(e-1)$

(c) 1

(d) $e-1$

(e) $2(e-1)$

5. $\int x\ln(x)\,dx =$?

(a) $\ln(x) + C$

(b) $x^2\ln(x) - \frac{1}{2}x^2 + C$

(c) $2x^2\ln(x) - x^2 + C$

(d) $\frac{1}{2}x^2\ln(x) - \frac{1}{4}x^2 + C$

(e) $\frac{1}{2}x^2\ln(x) + C$

6. If $h(x) = \int_0^x \sin(t^2+1)\,dt$, then $h'(1) =$?

(a) $\cos(2)$

(b) $-\cos(2)$

(c) $2\cos(2)$

(d) $-2\cos(2)$

(e) $\sin(2)$

7. $\int_0^1 (2x-1)^4 \, dx =?$

(a) 2

(b) 0

(c) 8

(d) 2/5

(e) 1/5

8. If f is a continuous function defined on the interval $[1,3]$, then $\int_1^3 f(x)\,dx - \int_2^3 f(x)\,dx =?$

(a) $-\int_2^3 f(x)\,dx$

(b) $-\int_1^2 f(x)\,dx$

(c) $-\int_1^3 f(x)\,dx$

(d) $\int_1^2 f(x)\,dx$

(e) $\int_2^3 f(x)\,dx$

9. $\int \sin(t)\cos^2(t)\,dt =?$

(a) $-1/3\cos^3(t) + C$

(b) $\sin^3(t) + C$

(c) $1/2\sin^2(t) \cdot 1/3\cos^3(t) + C$

(d) $\cos^3(t) - 2\sin^3(t) + C$

(e) $\sin(t) + C$

10. The sequence $\{a_n\}$, where $a_n = \dfrac{n+1}{2n+1}$,

(a) converges to 0.

(b) converges to 1/2.

(c) converges to 1.

(d) diverges.

(e) converges to 2.

11. The series $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{2n+1}$

(a) converges to 0.

(b) converges to 1/2.

(c) converges to 1.

(d) diverges.

(e) converges to 2.

12. Which of the following series converge?

$$\text{I.} \sum_{n=1}^{\infty} (-1)^n \qquad \text{II.} \sum_{n=1}^{\infty} \frac{1}{n^2+1} \qquad \text{III.} \sum_{n=1}^{\infty} \frac{1}{n+1}$$

(a) I, II, and III

(b) I and II

(c) II only

(d) II and III

(e) III only

13. The geometric series $2 - \frac{2}{3} + \frac{2}{9} - \frac{2}{27} + \cdots$

(a) converges to 3/2.

(b) converges to 3.

(c) converges to 1.

(d) converges to 0.

(e) diverges.

14. The following integral form appears in the Table of Integrals in your text:

$$\int \frac{du}{\sqrt{a^2 + u^2}} = \ln(u + \sqrt{a^2 + u^2}) + C$$

Use this form and an appropriate substitution to find $\int \frac{dx}{\sqrt{9 + 4x^2}}$.

(a) $\frac{1}{2} \ln(x + \sqrt{9 + x^2}) + C$

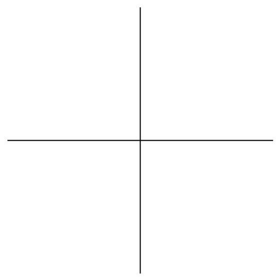
(b) $\frac{1}{2} \ln(x + \sqrt{9 + 4x^2}) + C$

(c) $\ln(x + \sqrt{9 + 4x^2}) + C$

(d) $\ln(2x + \sqrt{9 + 4x^2}) + C$

(e) $\frac{1}{2} \ln(2x + \sqrt{9 + 4x^2}) + C$

15. Consider the following graph of a function f:



The graph is a semicircle of radius 1. Then $\int_{-1}^{1} f(x)\,dx =$?

(a) π

(b) 2π

(c) 1

(d) $1/2$

(e) $\pi/2$

16. The power series $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ is the Maclaurin expansion of which of the following?

(a) e^x

(b) $\ln(x+1)$

(c) $\sin(x)$

(d) $\cos(x)$

(e) $\ln(x)$

Part II (MULTIPLE CHOICE, CALCULATORS ALLOWED).

1. The improper integral $\int_1^\infty \frac{dx}{x^{3/2}}$

(a) converges to 2.

(b) converges to 1.

(c) converges to 0.

(d) diverges to ∞.

(e) diverges to $-\infty$.

2. Let $f(x) = 2x^2 - 1$. Find the Riemann sum for f on the interval $[-1, 1]$, using 4 subintervals of equal width and taking the sample points to be the right endpoints of the subintervals.

(a) -1

(b) $-1/2$

(c) 0

(d) $-2/3$

(e) $1/2$

3. The speed of a runner increased steadily during the first two seconds of a race. The runner's speed at half-second intervals is given in the table. Based on the information given, which of the following is the best lower estimate for the distance traveled during these two seconds.

$t(s)$	0	0.5	1.0	1.5	2.0
$v(ft/s)$	0	6	10	13	17

(a) 31.5 ft

(b) 29 ft

(c) 14.5 ft

(d) 46 ft

(e) 23 ft

4. Which of the following is the best approximation to the length of the curve $y = x^3$, $0 \leq x \leq 2$? (Use your calculator to evaluate the integral involved.)

(a) 1.1207

(b) 0.8605

(c) 4.1701

(d) 8.6303

(e) 8.2462

5. A particle's velocity at any time t is given by $v(t) = \sin(t)$. Which of the following best approximates the average velocity of the particle for $0 \leq t \leq \pi/2$?

(a) 1 ft/s

(b) -1 ft/s

(c) 1.57 ft/s

(d) 0.64 ft/s

(e) 0 ft/s

6. A spring has a natural length of 0.2 m. If a 20-N force is required to keep it stretched to a length of 0.3 m, how much work is done in stretching the spring from its natural length to a length of 0.3 m?

(a) 0.5 J

(b) 1 J

(c) 0.05 J

(d) 20 J

(e) 10 J

7. Consider the series $\sum_{n=1}^{\infty} \frac{n+1}{n^2}$. Which of the following tests results in a successful determination of the convergence or divergence of this series?

(a) The alternating series test.

(b) The divergence test.

(c) The ratio test.

(d) Comparison with the terms of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

(e) Comparison with the terms of the series $\sum_{n=1}^{\infty} \frac{1}{n}$.

8. Which of the following is the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n^2}$?

(a) $(-1/2, 1/2]$

(b) $[-1/2, 1/2)$

(c) $[0, 1]$

(d) $(0, 1)$

(e) $[0, 2]$

9. In the partial fractions expansion $\frac{3x+1}{x^2-4} = \frac{A}{x+2} + \frac{B}{x-2}$, find B.

(a) $-5/4$

(b) $7/4$

(c) 1

(d) 3

(e) $1/4$

10. Find the x-coordinate of the centroid of the region in the first quadrant bounded by the coordinate axes and the curve $y = 1 - x^2$.

(a) 3/8

(b) 2/3

(c) 1/3

(d) 1/8

(e) 1/4

11. The region below the curve $y = \sqrt{x}$ and above the x-axis, for $0 \leq x \leq 2$, is rotated about the x-axis. Which of the following best approximates the volume of the resulting solid?

(a) 6.28

(b) 1.89

(c) 5.92

(d) 3.47

(e) 7.04

12. Let $f(x) = (2x - 1)^5$. What is the coefficient of x^3 is the Maclaurin expansion of f?

(a) 480

(b) 240

(c) 120

(d) 80

(e) 40

13. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n)}{n^3}$. Which of the following is true?

(a) The series diverges to ∞.

(b) The series diverges to $-\infty$.

(c) The series diverges, but not to ∞ or $-\infty$.

(d) The series converges absolutely.

(e) The series converges conditionally (but not absolutely).

14. Which of the following is correct for $-1 < x < 1$?

(a) $\ln(1+x) = 1 - x^2 + x^4 - x^6 + \cdots$

(b) $\ln(1+x) = 1 - x + x^2 - x^3 + \cdots$

(c) $\ln(1+x) = 1 + x + x^2 + x^3 + \cdots$

(d) $\ln(1+x) = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \cdots$

(e) $\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots$

Part III (FREE RESPONSE, CALCULATORS ALLOWED).

Note: Even though calculators are allowed, you must show your work in order to receive credit.

1. The region bounded by the curves $y = x^3$, $x = 1$, and $y = 0$ is rotated about the line $x = 1$ to form a solid.

(a) Use the "disk" method to set up a definite integral which will give the volume of the solid. You do not need to evaluate the integral.

(b) Now use the method of "cylindrical shells" to set up another definite integral which will also give the volume of the solid. You do not need to evaluate the integral.

2. A cylindrical tank sits on its base. The radius of the base is 3 m, and the tank is 10 m tall. The tank is full of water. Set up (but do not evaluate) a definite integral which gives the work done in pumping all the water out through a spout which is 2 m above the top of the tank. (The density of water is 1000 kg/m^3, and the acceleration due to gravity is 9.8 m/s^2.)

3. Recall the power series expansion $\sin(x) = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \cdots$.

(a) (5 points) Use the given expansion to derive a power series expansion for $\int_0^1 \sin(x^2)\, dx$. Show at least three nonzero terms.

(b) (5 points) Use the expansion in part (a) to approximate $\int_0^1 \sin(x^2)\, dx$ to within 0.0001.

4. (a) (5 points) Use the midpoint rule with $n = 4$ subintervals to approximate $\int_0^2 \ln(2x+1)$.

(b) Recall that the error estimate when the midpoint rule is used to approximate $\int_a^b f(x)\,dx$ is given by

$$|E_n| \leq K\frac{(b-a)^3}{24n^2},$$

where $|f''(x)| \leq K$ for $a \leq x \leq b$. Determine how large n must be chosen to ensure accuracy in the approximation in part (a) to within 0.001.

5. Use integration by parts to derive the reduction formula

$$\int x^n e^x \, dx = x^n e^x - n \int x^{n-1} e^x \, dx.$$

Answer Key.

Part I.

1. c
2. b
3. d
4. b
5. d
6. e
7. e
8. d
9. a
10. b
11. d
12. c
13. a
14. e
15. e
16. a

Part II.

1. a
2. b
3. c
4. d
5. d
6. b
7. e
8. c
9. b
10. a
11. a
12. d
13. d
14. e