# MATH 1242 – CALCULUS II    FALL 2018

# COMMON FINAL EXAMINATION

## UNC CHARLOTTE
### Department of Mathematics and Statistics

Last Name: _____    First Name: _____
(Please PRINT)                           (Please PRINT)

Student ID #: _____    Instructor: _____    Section: _____

For Grading Use Only:

| Problem | 1 | 2 | 3 | 4 | 5 |
|---------|---|---|---|---|---|
| Grade | | | | | |
| Out of | | | | | |

Free Response Score: _____

# PART III

- Part III consists of 5 free response problems. After you have handed in part I and your exam proctor announces that calculator may be used, you may use your calculator on this part of the exam. (Texas Instruments 83 or 84 or equivalent models of other brands are allowed. TI Inspire, TI 89 or equivalent calculators are NOT allowed on this exam.)
- Please show all of your work on the problem sheet provided. Work that is done on scratch paper or any other sheets will not be graded.
- You may use your calculator to check your answers, but complete justification must be shown for each problem. This includes all graphs, calculations and references to supporting theorems.
- Make sure that your name appears on each page of the test booklet.
- At the end of the exam you must hand in all test material including the test booklets, Opscan sheets and scratch paper.

**Part III (FREE RESPONSE, CALCULATORS ALLOWED).**

**Note: Even though calculators are allowed, you must show your work in order to receive credit.**

1. The region in the first quadrant bounded by the curves $y = 0$, $y = \sqrt{x}$, and $x = 1$ is rotated about the $y$-axis to form a solid.

(a) Use the method of cylindrical shells to set up a definite integral (which you need not evaluate) that gives the volume of this solid.

(b) Use the washer method to set up a definite integral (which you need not evaluate) that gives the volume of this solid.

2. Consider the region in the first quadrant bounded by the $x$-axis and the curve $y = \ln(x)$, $1 \leq x \leq 2$. Use calculus to find the $y$-coordinate of the centroid of this region. Round your answer to 3 decimal places. You may use your calculator to evaluate any integrals involved, but you must set up the integrals.

3. Consider the definite integral $\int_0^1 e^{-2x} \, dx$.

(a) Use the Midpoint Rule with $n = 2$ subintervals of equal width to approximate the definite integral above. Round your answer to two decimal places. Be sure to show your work!

(b) If $|f''(x)| \leq K$ for $a \leq x \leq b$, and $E_M$ is the error when the midpoint rule with $n$ subintervals is used to approximate $\int_a^b f(x) \, dx$, then

$$|E_M| \leq \frac{K(b-a)^3}{24n^2}.$$

Use this fact to give an upper bound on the error in your approximation in part (a).

4. Consider the following power series expansion: $e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \cdots$.

(a) Use the first 3 terms of the expansion to approximate $\int_0^{0.5} e^{-x^2}\, dx$. Round your answer to 4 decimal places.

(b) Use the fact that the series expansion for $\int_0^{0.5} e^{-x^2}\, dx$ is alternating to give an upper bound on the error of you approximation in part (a).

5. A spherical tank with a radius of 1 m is buried with the top of the tank 3 m below ground level. The tank is full of fuel oil with density 800 kg/m$^3$. Set up (but you need not evaluate) a definite integral that gives the work done in pumping all of the oil out of the tank at ground level. (The gravitational constant is 9.8 m/s$^2$.)