MATH 1242 – CALCULUS II
COMMON FINAL EXAMINATION

Fall 2019

UNC CHARLOTTE
Department of Mathematics and Statistics

Last Name: ______________________ First Name: ______________________
(Please PRINT) (Please PRINT)

Student ID #: ________________ Instructor: ________________ Section: ________

For Grading Use Only:

Problem	1	2	3	4	5
Grade					
Out of					

Free Response Score ________

PART III

- Part III consists of 5 free response problems. After you have handed in part I and your exam proctor announces that calculator may be used, you may use your calculator on this part of the exam. (Texas Instruments 83 or 84 or equivalent models of other brands are allowed. TI Inspire, TI 89 or equivalent calculators are NOT allowed on this exam.)
- Please show all of your work on the problem sheet provided. Work that is done on scratch paper or any other sheets will not be graded.
- You may use your calculator to check your answers, but complete justification must be shown for each problem. This includes all graphs, calculations and references to supporting theorems.
- Make sure that your name appears on each page of the test booklet.
- At the end of the exam you must hand in all test material including the test booklets, Opscan sheets and scratch paper.

Part III (FREE RESPONSE, CALCULATORS ALLOWED).

Note: Even though calculators are allowed, you must show your work in order to receive credit.

1. Consider the region in the first quadrant under the graph of $y = \cos(x)$, $0 \le x \le \pi/2$. In each case below, sketch the solid formed when the region is rotated about the given line. Then set up a definite integral that gives the volume of the solid. Use your calculator to evaluate the integral, and round your answer to 3 decimal places.

(a) The line $y = 0$ (the x-axis). Use the washer method.

(b) The line $x = \pi$. Use the method of cylindrical shells.

2. Consider the region in the first quadrant under the curve $y = 1 - x^2$, $0 \leq x \leq 1$. Use calculus to find the x-coordinate $\bar{x}$ of the centroid of the region. Set up the necessary integrals, and evaluate them by hand.

3. Consider the Maclaurin expansion $\cos(x^2) = 1 - \frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \cdots$, valid for all real numbers x.

(a) Use this expansion to give the first three terms of a power series expansion for $\int \cos(x^2)\,dx$.

(b) Use your expansion in part (a) to approximate $\int_0^{0.9} \cos(x^2)\,dx$. Again, use three terms. Round your answer to four decimal places.

(c) Use the Alternating Series Estimation Theorem to give an upper bound on the error in your estimate in part (b).

4. A right cylindrical tank sits on its base. The base radius of the tank is 3 m, and tank is 5 m tall. The tank is filled with sludge. The density of the sludge is not constant; rather, the density is given by $\rho(x) = (x + 32)^2 \text{ kg/m}^3$, where x is the distance from the top of the tank. Set up, and use your calculator to evaluate, a definite integral that gives the work W done in pumping the sludge out at the top of the tank. The gravitational constant is $g = 9.8\,\text{m/s}^2$.

5. Consider the definite integral $I = \int_0^1 \frac{1}{x+1}\, dx$.

(a) Use the Midpoint Rule with $n = 2$ subintervals of equal width to approximate the definite integral above. Round your answer to two decimal places. Be sure to show your work!

(b) If $|f''(x)| \leq K$ for $a \leq x \leq b$, and E_M is the error when the midpoint rule with n subintervals is used to approximate $\int_a^b f(x)\, dx$, then

$$|E_M| \leq \frac{K(b-a)^3}{24n^2}.$$

Determine how large n must be to guarantee that the approximation using the Midpoint Rule with n subintervals has error less that 0.00001.