

Name \_\_\_\_\_  
Student ID # \_\_\_\_\_

Instructor: \_\_\_\_\_  
Section/Time \_\_\_\_\_

This exam is divided into three parts. **Calculators are not allowed on Part I. Use of cell phones/smart phones is prohibited at all times.** You have three hours for the entire test, but you have only one hour to finish Part I. You may start working on the other two parts of the exam whenever you are done with Part I, but you cannot use your calculator until ALL of the Part I answer sheets are collected. After these answer sheets are collected, your instructor will announce that calculators are allowed on Parts II and III.

These pages contain Part I which consists of 13 multiple choice questions. These questions must be answered without the use of a calculator.

- You must use a pencil with a soft black lead (#2 or HB) to enter your answers on the answer sheet
- For each question choose the response which best fits the question
- If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.
- There is no penalty for guessing
- If you mark more than one answer to a question, the question will be scored as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.
- Make sure that your name appears on the answer sheet and that you fill in the circles corresponding to your name.**

**After one hour, you MUST hand in the answer sheet for Part I.  
At the end of the exam, you MUST hand in all remaining test materials including test booklets, answer sheet for Part II, and scratch paper.**

1. Let  $f(x) = x^3 + \frac{8}{x} + 4$ . Evaluate  $f'(2)$ .

- (a) 2
- (b) 4
- (c) 6
- (d) 8
- (e) 10

2. Let  $g(x) = (x^2 + 1)^3$ . Evaluate  $g'(1)$ .

- (a) 8
- (b) 12
- (c) 18
- (d) 24
- (e) 27

3. Let  $h(x) = \frac{3x-1}{x^2+1}$ . Evaluate  $h'(x)$ . (Simplify the numerator.)

- (a)  $\frac{3}{2x}$
- (b)  $\frac{-6x^2+2x}{(x^2+1)^2}$
- (c)  $\frac{-6x^2+2x+3}{(x^2+1)^2}$
- (d)  $\frac{-3x^2+2x+3}{(x^2+1)^2}$
- (e)  $\frac{9x^2-2x+3}{(x^2+1)^2}$

4. Let  $F(x) = e^{2x} \cos(3x)$ . Evaluate  $F'(x)$ .

- (a)  $-e^{2x} \sin(3x)$
- (b)  $-6e^{2x} \sin(3x)$
- (c)  $e^{2x}(-3\sin(3x) + 2\cos(3x))$
- (d)  $e^{2x}(-2\sin(3x) + 3\cos(3x))$
- (e)  $e^{2x}(-\sin(3x) + \cos(3x))$

5. Let  $f(x) = \arcsin(\ln(x))$ . Evaluate  $f'(x)$ . (The arcsin is the inverse sine function. You could also write  $f(x) = \sin^{-1}(\ln(x))$ .)

- (a)  $\frac{1}{\sqrt{1-(\ln(x))^2}}$
- (b)  $\frac{1}{x\sqrt{1-(\ln(x))^2}}$
- (c)  $\frac{1}{\sqrt{1-x^2}}$
- (d)  $\frac{1}{x\sqrt{1-x^2}}$
- (e)  $\frac{1}{x\sqrt{x^2-1}}$

6. Which formula below defines the derivative of  $f(x)$  at the point  $x = 2$ ?

- (a)  $\lim_{h \rightarrow 2} \frac{f(2) - f(h)}{h}$
- (b)  $\lim_{h \rightarrow 2} \frac{f(x+h) - f(2)}{h}$
- (c)  $\lim_{h \rightarrow 0} \frac{f(2+h) - 2}{h}$
- (d)  $\lim_{h \rightarrow 0} \frac{f(2)+h}{h}$
- (e)  $\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$

7. The graph of  $g(x) = \frac{1}{2}x^4 - 2x^3 - 9x^2$  is concave down on the interval

- (a)  $(-\infty, 3)$
- (b)  $(3, \infty)$
- (c)  $(-1, 3)$
- (d)  $(-\infty, 2)$
- (e)  $(2, \infty)$

8. Evaluate  $\lim_{x \rightarrow 2} \frac{x^2 + 7x - 18}{x - 2}$

- (a) The limit does not exist.
- (b)  $-9$
- (c)  $0$
- (d)  $7$
- (e)  $11$

9. Evaluate  $\lim_{x \rightarrow \infty} \frac{3x - 2}{x^2 + 4x + 4}$

- (a)  $-2$
- (b)  $0$
- (c)  $2$
- (d)  $3$
- (e) The limit does not exist

10. Use L'Hospital's rule to evaluate  $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x}$

- (a) 0
- (b) 1
- (c) 2
- (d) 4
- (e) The limit does not exist.

11. Find the general antiderivative of  $f(x) = \frac{3}{x^2} + \cos(x)$

- (a)  $3 \cdot \ln|x^2| + \sin(x) + C$
- (b)  $3 \cdot \ln|x^2| - \sin(x) + C$
- (c)  $\frac{3}{x} - \sin(x) + C$
- (d)  $-\frac{3}{x} + \sin(x) + C$
- (e)  $\frac{9}{x^3} + \sin(x) + C$

12. Find the general antiderivative of  $g(x) = 2x(x + 1)$

- (a)  $\frac{1}{2}x^4 + C$
- (b)  $x^2(\frac{1}{2}x^2 + x)$
- (c)  $2 + C$
- (d)  $\frac{2}{3}x^3 + x^2 + C$
- (e) The function  $g$  does not have an antiderivative.

13. Suppose  $f(x) = 2e^x + 1$ . Then  $f$  is an invertible function. Find a formula for its inverse.

- (a)  $f^{-1}(x) = \frac{1}{2e^x + 1}$
- (b)  $f^{-1}(x) = \frac{2}{e^x} - 1$
- (c)  $f^{-1}(x) = \frac{x-1}{2e}$
- (d)  $f^{-1}(x) = \ln\left(\frac{x-1}{2}\right)$
- (e)  $f^{-1}(x) = \ln(2e^x + 1)$

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These pages contain Part II which consists of 12 multiple choice questions. After the answer sheets for Part I have all been collected, and your instructor announces that calculators are OK, you are allowed to use a calculator on this part of the exam. **Use of cell phones/smart phones is prohibited at all times.**

- You must use a pencil with a soft black lead (#2 or HB) to enter your answers on the answer sheet
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- If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.
- There is no penalty for guessing
- If you mark more than one answer to a question, the question will be scored as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.
- **Make sure that your name appears on the answer sheet for Part II and that you fill in the circles corresponding to your name.**

**At the end of the exam, you MUST hand in all remaining test materials including test booklets, answer sheet for Part II, and scratch paper.**

1. Let  $f(x) = x^2g(x)$ . Suppose that  $g(2) = 3$ , and  $g'(2) = -1$ . Evaluate  $f'(2)$ .

- (a)  $-4$
- (b)  $0$
- (c)  $4$
- (d)  $8$
- (e)  $12$

2. Let  $f(x) = \begin{cases} x^2+1, & x < 1 \\ 3, & x = 1 \\ 3x-1, & x > 1 \end{cases}$  Evaluate  $\lim_{x \rightarrow 1} f(x)$

- (a)  $0$
- (b)  $1$
- (c)  $2$
- (d)  $3$
- (e) The limit does not exist

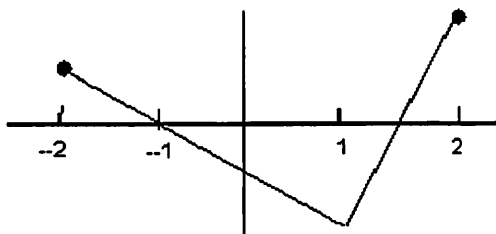
3. Suppose that  $f$  is a differentiable function such that  $f(1) = 5$  and  $f'(x) \leq 2$  for all  $x$ . Use the mean value theorem to decide what the largest possible value of  $f(4)$  is.

- (a)  $11$
- (b)  $10$
- (c)  $8$
- (d)  $7$
- (e) There is no largest value

4. Suppose that the derivative of a function  $f$  is given by  $f'(x) = x^2(x-1)(x-2)$ . Which of the following statements is correct? (Note that the statements are about the original function  $f$  and not the derivative.)

- (a) The function  $f$  has a local maximum at  $x = 0$
- (b) The function  $f$  has a local minimum at  $x = 0$
- (c) The function  $f$  has a local maximum at  $x = 2$
- (d) The function  $f$  has a local minimum at  $x = 2$
- (e) The function  $f$  has a local minimum at  $x = 1$ .

5. The complete graph of  $y = f'(x)$  is shown. On which interval(s) below is  $f$  concave up? Note that you are given the graph of the derivative  $f'$ , but you are being asked about the function  $f$ .



- (a)  $(-2, -1) \cup (1.5, 2)$   
 (b)  $(-1, 1.5)$   
 (c)  $(-2, 1)$   
 (d)  $(1, 2)$   
 (e)  $f$  is never concave up

6. Let  $f(x) = \begin{cases} \frac{x^2-1}{x-1}, & x \neq 1 \\ k, & x = 1 \end{cases}$ .

Choose  $k$  so that the function  $f$  is continuous at  $x = 1$ .

- (a)  $k = -2$   
 (b)  $k = -1$   
 (c)  $k = 0$   
 (d)  $k = 1$   
 (e)  $k = 2$

7. Three functions are listed below. Which of these functions have an inverse function on the domain  $(-\infty, \infty)$ ?

(I)  $f(x) = x^2$       (II)  $g(x) = 3x + 1$       (III)  $h(x) = \sin(x)$ .

- (a) only  $f(x)$   
 (b) only  $g(x)$   
 (c) only  $h(x)$   
 (d) both  $g(x)$  and  $h(x)$   
 (e) both  $f(x)$  and  $g(x)$

8. Suppose that  $x$  and  $y$  are functions of  $t$ , that  $x^2 + y^2 = 25$ , and that  $\frac{dy}{dt}$  is always 6. Find  $\frac{dx}{dt}$  when  $y = 4$  and  $x > 0$ .

- (a)  $-8$   
 (b)  $-6$   
 (c)  $-4$   
 (d)  $2$   
 (e)  $4$

9. A colony of bacteria grows at a rate proportional to its size. Initially, there are 100 cells in the colony. The colony triples in size every 2 hours. Approximately how many cells are present after 7 hours?
- (a) 3528  
 (b) 3981  
 (c) 4217  
 (d) 4677  
 (e) 5103

10. Find the linear approximation  $L(x)$  to  $f(x) = \frac{1+x^2}{4}$  at the point  $x = 1$ .

- (a)  $L(x) = -2x$   
 (b)  $L(x) = 4 - 4x$   
 (c)  $L(x) = 2 - 2x$   
 (d)  $L(x) = 6 - 4x$   
 (e)  $L(x) = 4 - 2x$

11. Let  $xy^3 + 3x - 2y = 6$ . Find  $\frac{dy}{dx}$  at the point  $(2, 1)$ .

(a)  $-1$   
 (b)  $-\frac{3}{2}$   
 (c)  $-\frac{1}{2}$   
 (d)  $-\frac{1}{3}$   
 (e)  $-\frac{1}{4}$

12. We wish to solve  $x^3 + 3x = 5$  using Newton's method. Use  $x_1 = 1$  as your initial approximation and find  $x_2$ , the second approximation. (You are not being asked for the exact solution.) Round your answer to two decimal places.

(a) 1.15  
 (b) 1.17  
 (c) 1.19  
 (d) 1.21  
 (e) 1.23



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These pages contain Part III which consists of 5 free response questions. Calculators are permitted on this part of the exam. **Use of cell phones/smart phones is prohibited at all times.**

Please show all your work in this test booklet. Loose paper will not be graded.

•If you are basing your answer on a graph on your calculator, sketch this graph in the answer booklet. Be sure to label your window by putting a scale on the axes.

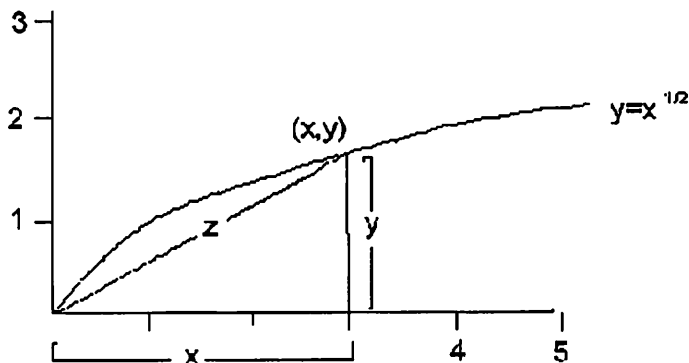
•Make sure that your name appears on each page.

**At the end of the exam, you MUST hand in all remaining test materials including test booklets, answer sheet, and scratch paper.**

PROBLEM	1	2	3	4	5
GRADE					

**FREE RESPONSE SCORE:** \_\_\_\_\_

1. A particle is moving along the curve  $y = \sqrt{x}$ . As the particle passes through the point  $(4, 2)$ , its  $x$ -coordinate is increasing at a rate of 5 cm per second. How fast is the distance from the particle to the origin changing at this instant?



(a) Suppose that the particle is located at a point  $(x, y)$  somewhere on the curve  $y = \sqrt{x}$ . Let  $z$  represent the distance from the particle to the origin (see the diagram above). Write an equation that relates  $x$  and  $y$  to  $z$ .

(b) When the point  $(x, y)$  is on the curve, how is  $y$  related to  $x$ ? Use this fact and the equation in part (a) so that you can express  $z$  only in terms of  $x$  (eliminate  $y$ ).

(c) Differentiate the equation you obtained in part (b) with respect to the time  $t$ .

(d) What is  $\frac{dx}{dt}$  when the particle is at the point  $(4, 2)$ ? (Reread the original question in bold at the top of this page.) Use this fact and (c) to evaluate  $\frac{dz}{dt}$  when the particle is at the point  $(4, 2)$ .

2. A function  $f$  satisfies the conditions below. Use this information to answer the questions below.

The domain of  $f$  is all real numbers except  $x = 0$ .

$$f(-1) = -1, f(1) = 0, f\left(\frac{3}{2}\right) = -\frac{1}{2}.$$

$x = 0$  is a vertical asymptote

$$\lim_{x \rightarrow \infty} f(x) = -1$$

$f' > 0$  on the intervals  $(-1, 0)$  and  $(0, 1)$

$f' < 0$  on the intervals  $(-\infty, -1)$  and  $(1, \infty)$

$f'' < 0$  on the interval  $(0, \frac{3}{2})$

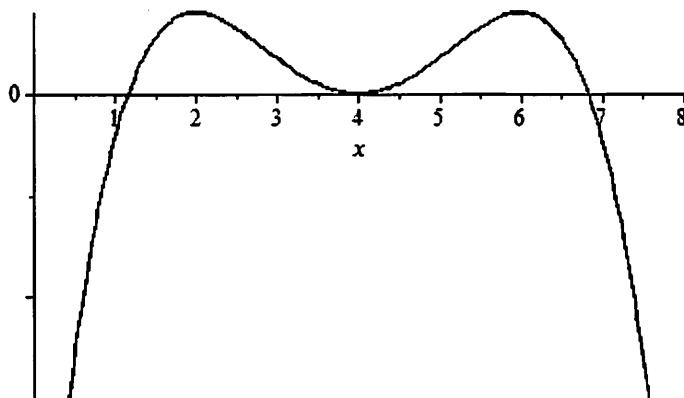
$f'' > 0$  on the intervals  $(-\infty, 0)$  and  $(\frac{3}{2}, \infty)$

(a) List any intervals where the graph of  $y = f(x)$  is concave up.

(b) List any horizontal asymptotes.

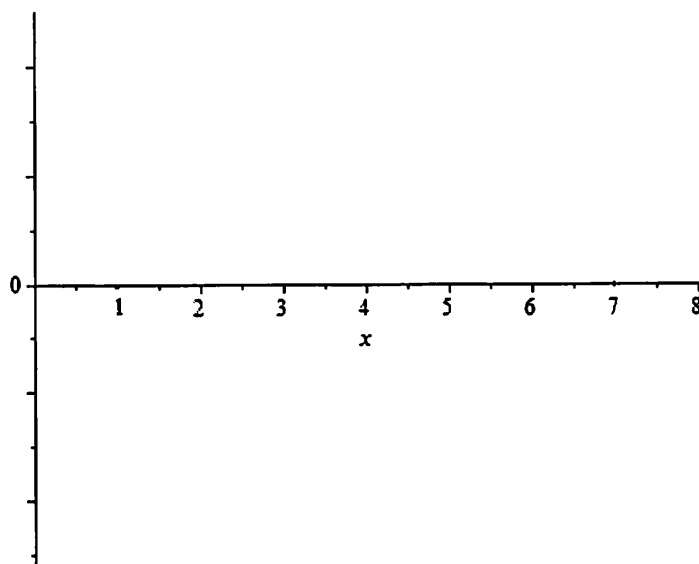
(c) Sketch a graph of the function  $f(x)$ .

3.



The curve shown is the graph of  $y = f(x)$ .

- List the interval(s) where  $f$  is decreasing.
- List the value(s) of  $x$  where  $f'(x) = 0$
- List an interval where  $f$  is concave up.
- Sketch a graph of the derivative  $f'(x)$ .



4. Let  $f(x) = x^3 e^{-4x}$

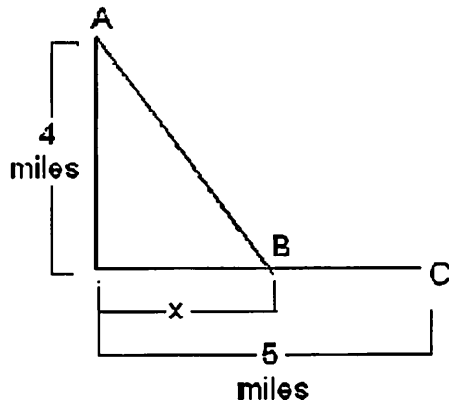
(a) Find  $f'(x)$

(b) Factor the expression in part (a) to find the critical numbers for  $f$ .

(c) Determine the interval(s) where  $f$  is increasing and the interval(s) where  $f$  is decreasing. Show your work.

(d) At each critical number, determine whether  $f$  has a relative maximum (same as local maximum), a relative minimum (same as local minimum), or neither.

5. Stephanie plans to warm up by walking from A to B (see the diagram) at 3 miles per hour and then power walk from B to C at 5 miles per hour. She wants to choose  $x$  so that the total time to travel from A to C is a minimum. (Recall that distance equals rate times time.)



- (a) Express the time it takes to walk from A to B at 3mph in terms of  $x$ .
- (b) Express the time it takes to walk from B to C at 5mph in terms of  $x$ .
- (c) Add the times in parts (a) and (b) to get the total trip time.
- (d) Use calculus to find the value of  $x$  that minimizes the total trip time in part (c) Show your work.