MATH 1241 FINAL EXAM SPRING 2015

Part I, No Calculators Allowed

1. Evaluate the limit:

$$\lim_{X \to -10} (2X - 10)$$

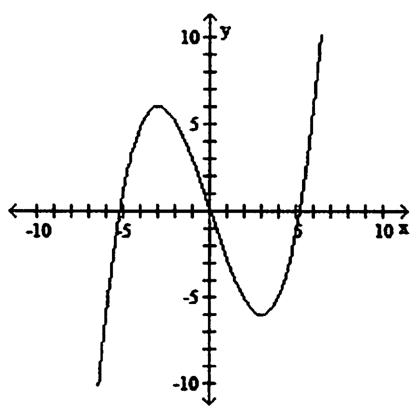
a) 10
b) -30
c) -10
d) 30
e) 0

2. Evaluate the limit:

$$\lim_{X \to 3} \frac{X^2 - 8X + 15}{X - 3}$$

a) -5
b) 6
c) 14
d) -2
e) 0

3. Use the graph of the function f(x) to locate the local extrema and identify the intervals where the function is concave up and concave down.



a) Local min at x = 3; local max at x = -3 ; concave down on (0, ∞); concave up on (-∞, 0)
b) Local min at x = 3; local max at x = -3 ; concave up on (−∞, -3) and (3, ∞); concave down on (-3, 3)
c) Local min at x = 3; local max at x = -3 ; concave up on (0, ∞); concave down on (-∞, 0)
d) Local max at x = 3; local min at x = -3 ; concave up on (−∞, -3) and (3, ∞); concave down on (-3, 3)
e) Local min at x = -3; local max at x = 3 ; concave up on (0, ∞); concave down on (-∞, 0)

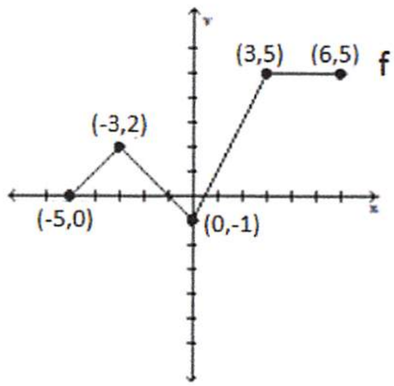
4. Which of the following is a derivative of $f(t) = 2t^2 + 7t + 4$.
   a) $2t + 7$
   b) $4t + 7$
   c) $4t^2 + 7$
   d) $2t^2 + 7$
   e) $2t^2 + 7 + 1$

5. Find an equation for the line that is tangent to the graph of $g(x) = \frac{x^3}{2}$ at the point (2,4).
   a) $y = 2x - 8$
   b) $y = 8x + 6$
   c) $y = 6x - 8$
   d) $y = 2x + 8$
   e) $y = 6x - 2$

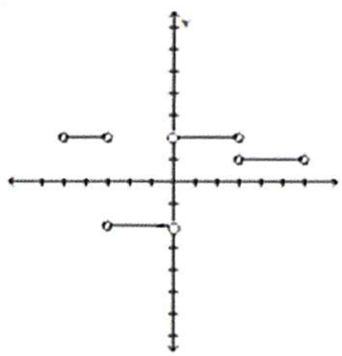
6. Which of the following is the derivative of $f(x) = x^3 \ln x$
   a) $x^2 \ln x + x^2$
   b) $(3x^2 + 1)\ln x$
   c) $3x^2 \ln x$
   d) $3x^2 \ln x + x^2$
   e) None of these

7. Which of the following is the derivative of $g(x) = \frac{x^3}{x-1}$
   a) $\frac{2x^3-3x^2}{(x-1)^2}$
   b) $\frac{-2x^3+3x^2}{(x-1)^2}$
   c) $\frac{2x^3+3x^2}{(x-1)^2}$
   d) $\frac{2x^3-3x^2}{(x+1)^2}$
   e) $\frac{-2x^3-3x^2}{(x-1)^2}$

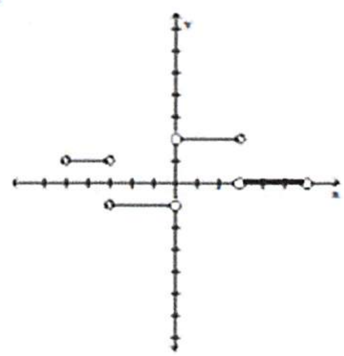
8. The graph of y = f(x) in the accompanying figure below is made of line segments joined end to end. Which answer choice represents the graph the derivative of f(x)?



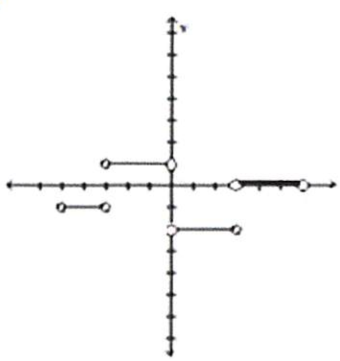
A)



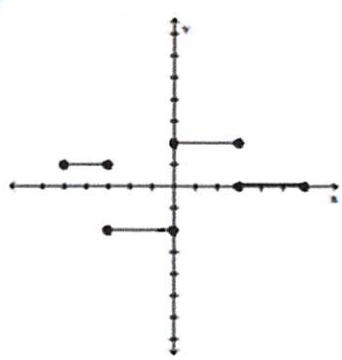
B)



C)



D)



E) None of these

9. Which of the following is the derivative of $h(x) = (6e^{2x} - x)^3$.

a) $3(6e^{2x} - x)^2(12e^{2x})$
b) $3(12xe^{2x-1} - 1)^2$
c) $3(12e^{x} - 1)^2$
d) $3(6e^{2x} - x)^2(12e^{2x} - 1)$
e) $3(6e^{2x} - x)^2$

10. Which of the following in the derivative of $f(t) = \cos(\sqrt{8t + 11})$

a) $-\sin(\sqrt{8t + 11})$
b) $\sin(\sqrt{8t + 11})$
c) $\frac{-1}{2\sqrt{8t+11}}\sin(\sqrt{8t + 11})$
d) $\frac{4}{\sqrt{8t+11}}\sin(\sqrt{8t + 11})$
e) $\frac{-4}{\sqrt{8t+11}}\sin(\sqrt{8t + 11})$

11. Evaluate the limit

$$\lim_{x\to\infty}\sqrt{\frac{36x^2 + x - 3}{(x - 13)(x + 1)}}$$

a) ∞
b) 36
c) 6
d) 0
e) 1

12. Let $f(x) = x^2 + 3x + 2$ and consider the interval [1,2]. Find the values of c that satisfy the equation $\frac{f(b)-f(a)}{b-a} = f'(c)$ in the conclusion of the Mean Value Theorem.

a) $\frac{2}{3}$

b) $1, 2$

c) $0, \frac{3}{2}$

d) $0, \frac{2}{3}$

e) $\frac{3}{2}$

13. Which of the following is the derivative of $h(x) = \left(1 - \frac{1}{x^3}\right)^{-1}$

a) $-\left(1 - \frac{1}{x^3}\right)^{-2}$

b) $-\frac{3}{x^4}\left(1 - \frac{1}{x^3}\right)^{-2}$

c) $-\frac{3}{x^2}\left(1 - \frac{1}{x^3}\right)^{-2}$

d) $\frac{3}{x^4}\left(1 - \frac{1}{x^3}\right)^{-2}$

e) $-\left(\frac{3}{x^4}\right)^{-2}$

14. Let $2xy - y^2 = 1$. Use implicit differentiation to find $\frac{dy}{dx}$.

a) $\frac{x}{x-y}$

b) $\frac{x}{y-x}$

c) $\frac{y}{y-x}$

d) $\frac{y}{x-y}$

e) None of these

15. Find the second derivative of the function $y = \left(9 + \frac{2}{x}\right)^4$.

a) $12\left(9 + \frac{2}{x}\right)^2$

b) $8\left(9 + \frac{2}{x}\right)^3$

c) $-\frac{8}{x^2}\left(9 + \frac{2}{x}\right)^3$

d) $\frac{48}{x^4}\left(9 + \frac{2}{x}\right)^2 + \frac{16}{x^3}\left(9 + \frac{2}{x}\right)^3$

e) $-\frac{24}{x^2}\left(9 + \frac{2}{x}\right)^2 + \frac{16}{x^3}\left(9 + \frac{2}{x}\right)^3$

16. Find the general antiderivative of the function $f(x) = \sqrt{x} + \cos(x)$.

a) $\frac{1}{2\sqrt{x}} + \sin(x) + C$

b) $\frac{1}{2\sqrt{x}} - \sin(x) + C$

c) $\frac{2}{3}x^{\frac{3}{2}} - \sin x + C$

d) $\frac{2}{3}x^{\frac{3}{2}} + \sin(x) + C$

e) $\sqrt{\frac{x^3}{2}} + \cos\left(\frac{x^2}{2}\right) + C$

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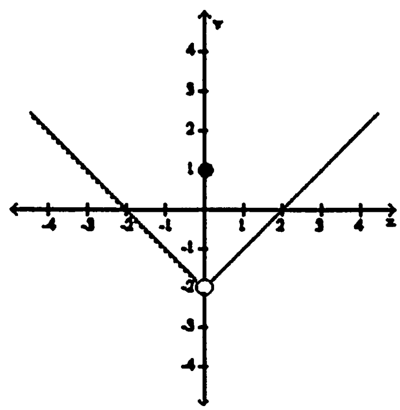
Part II, Calculators Allowed

1. Suppose $f(x)$ is given by the graph below. Evaluate $\lim_{x \to 0} f(x)$.



   a) 0
   b) 1
   c) -2
   d) -1
   e) Does not exist

2. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when x =

   a) -1 only
   b) 2 only
   c) -1 and 0 only
   d) -1 and 2 only
   e) -1, 0, and 2 only

3. Suppose that the functions f and g and their derivatives with respect to x have the following values at the given values of x. Let $h(x) = f(g(x))$. Find $h'(4)$.

x	f(x)	g(x)	f '(x)	g '(x)
3	1	4	6	5
4	-3	3	5	-4

a) -20
b) 6
c) 1
d) -24
e) 18

4. The radius of a circular disk is given as 24 cm with a possible error of no more than 0.2 cm. Use differentials to estimate the maximum error in the calculated area of the disk.

a) 15.08 cm^2
b) 48 cm^2
c) 9.6 cm^2
d) 30.16 cm^2
e) 4.8 cm^2

5. Find the point or points (x, y) on the graph of $f(x) = 2x^2 - 3x$ with tangent lines parallel to the line $y = 9x + 9$.

a) (3,9)
b) (0,0), (3,9)
c) (6,9)
d) (0, 0), (6,9)
e) (3, 18)

6. The function $g(x)$ has a derivative for each value of x and $f(x) = \sqrt{g(x)}$. Find $f'(3)$ given that $g(3) = 9$ and $g'(3) = 6$.

   a) $f'(3) = 0.2$
   b) $f'(3) = 1$
   c) $f'(3) = 1.5$
   d) $f'(3) = 0.7$
   e) $f'(3)$ does not exist

7. A watermelon dropped from rest from a tall building, falls $y = 16t^2$ ft in t seconds. Find the watermelon's average speed during the first 4 seconds of its fall.

   a) 65 ft/sec
   b) 128 ft/sec
   c) 64 ft/sec
   d) 32 ft/sec
   e) 144 ft/sec

8. For a motorcycle traveling at speed v (in mph) when the brakes are applied, the distance d (in feet) required to stop the motorcycle may be approximated by the formula $d(v) = 0.05v^2 + v$. Find the instantaneous rate of change of distance with respect to velocity when the speed is 41 mph.

   a) 5.1 mph
   b) 10.2 mph
   c) 42 mph
   d) 4.1 mph
   e) 4.2 mph

9. Let f and g be differentiable functions with the following properties:

(i) $g(x) > 0$ *for all x*
(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

a) $f'(x)$
b) $g(x)$
c) e^x
d) 0
e) 1

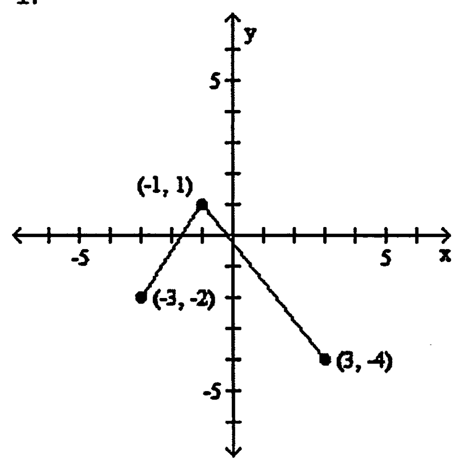
10. If $f(x) = \ln|x^2 - 1|$, then $f'(x) =$

a) $\left|\frac{2x}{x^2-1}\right|$
b) $\frac{2x}{|x^2-1|}$
c) $\frac{2|x|}{x^2-1}$
d) $\frac{2x}{x^2-1}$
e) $\frac{1}{x^2-1}$

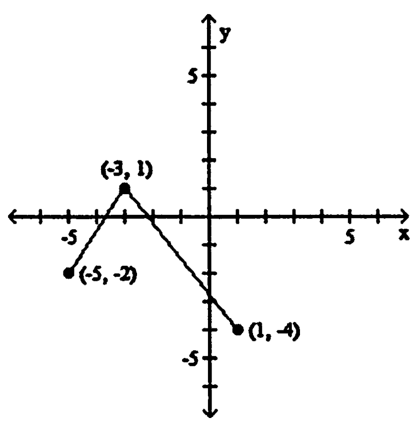
11. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x\to 2} f(x)$ is

a) ln 2
b) ln 8
c) ln 16
d) 4
e) Does not exist

12. The graph of a function $y = f(x)$ is given below. Select the answer choice that represents the graph of $f(x+2)-1$.



A)



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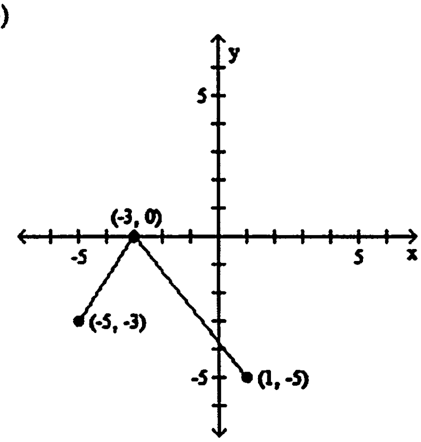
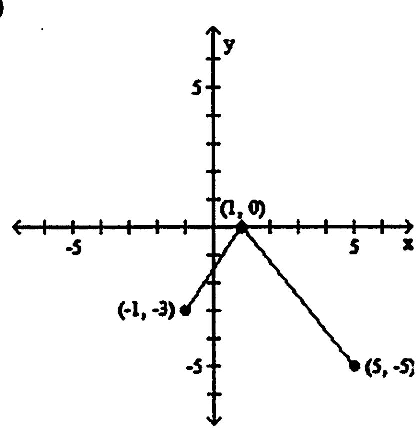
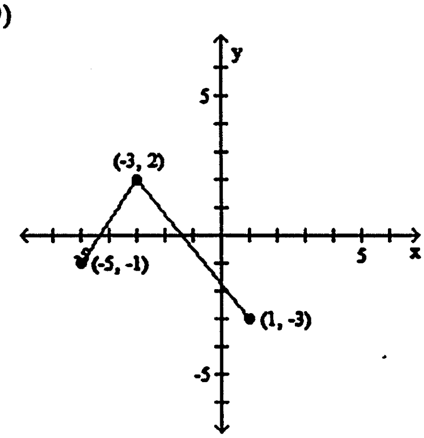


C)



D)

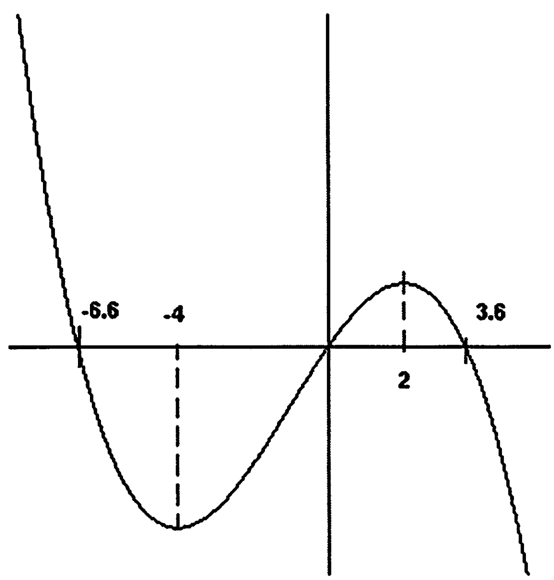


E) None of these

13. The linear approximation L(x) of the function $f(x) = \sqrt{3x + 64}$ at the point $x = 0$ is

a) $L(x) = \frac{3}{8}x + 8$
b) $L(x) = \frac{3}{16}x + 8$
c) $L(x) = \frac{3}{16}x - 8$
d) $L(x) = \frac{3}{8}x - 8$
e) $L(x) = -\frac{3}{8}x - 8$

14. The graph of $y = f'(x)$ is shown. Over which of the following interval(s) is f concave up? (Note that you were given the graph of the derivative, f', but are being asked about f.)



a) $(-\infty, 0)$
b) $(-\infty, -6.6) \cup (0, 3.6)$
c) $(-6.6, 0)$
d) $(-4, 2)$
e) $(-\infty, -4) \cup (2, \infty)$

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Part III, Calculators Allowed

1. Let $f(x) = \frac{x}{1+x^2}$.

a) What is the domain of the function $f(x)$? Write your answer in interval notation.

b) Find any horizontal or vertical asymptotes.

Vertical:	
Horizontal:	

c) Calculate the derivative $f'(x)$.

d) Find all the critical points. Where is the function f(x) increasing and where is it decreasing? Show your work. Write your answer in interval notation.

Critical Points:	
Increasing:	
Decreasing:	

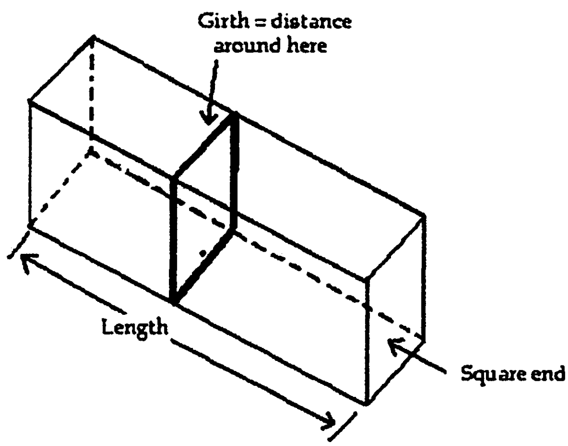
2. The world population was 2,560 million in 1950 and 3,040 million in 1960. Assume the growth rate of the population is proportional to the size of the population $P(t)$. Also assume that $t = 0$ in 1950.

   a) Write down the population model as an exponential function of time. Explicitly, calculate all the parameters involved in the model using the data provided above.

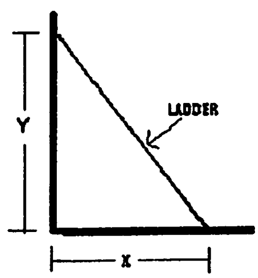
   b) What is the predicted world population in 2040?

   c) When is the world population predicted to reach 10 billion?

3. A private shipping company will accept a box for domestic shipment only if the sum of its length and girth (distance around) does not exceed 120 in. What dimensions will give a box with a square end the largest possible volume? (You must show your work!)



4. A 20 foot ladder rests against a vertical wall. Let y be the distance between the top of the ladder and the ground and let x be the distance from the bottom of the ladder to the wall. If the bottom of the ladder slides away from the wall at a constant rate of 2 feet per second, how fast is the top of the ladder of the sliding down the wall when the bottom of the ladder is 16 feet from the wall?



5.

a) Evaluate $\lim_{x\to\infty} \frac{\ln x}{\sqrt[3]{x}}$

b) Evaluate $\lim_{x\to 1} \frac{1 - x + \ln x}{1 + \cos(\pi x)}$

c) Let $f(x) = 2x^2 + 6x$. Use the definition of derivative $f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$ to evaluate $f'(3)$. You must show your work. (No credit will be given for answers based on the fast rules.