MATH 1241– CALCULUS I SPRING 2016

COMMON FINAL EXAMINATION

UNC CHARLOTTE
Department of Mathematics and Statistics

Last Name: _____ First Name: _____
(Please PRINT) (Please PRINT)

Student ID #: _____ Instructor: _____ Section: _____

This exam is divided into three parts. **Calculators are not allowed on Part I.** You have 3 hours for the entire exam, but you have only one hour to finish Part I. You may start working on the other two parts of the test after the first hour, but you cannot use your calculator during this time. You may use your calculator ONLY after your exam proctor has announced that calculators are allowed on Parts II and III. (Texas Instruments 83, 84, 89 or equivalent models of other brands are allowed. **TI Inspire**, TI 92 or equivalent calculators are NOT allowed at all on this exam.)

PART I

- Part I consists of 14 multiple choice problems. These problems must be answered without the use of a calculator.
- You must use a pencil with soft black lead (#2 or HB) to indicate your answers on the Opscan sheets.
- For each question, choose the response which best fits the question.
- If you wish to change an answer, make sure that you completely erase your old answer and any extraneous marks.
- There is no penalty for guessing.
- If you mark more than one answer to a question, that question will be marked as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the Opscan sheets.
- Make sure that your name appears on the Opscan sheets and that you fill in the circles corresponding to your name in the format Last, First.
- At the end of the exam you must hand in all test material including the test booklets, Opscan sheets and scratch paper.

MATH 1241 FINAL EXAM SPRING 2016
Part I, No Calculators Allowed

1. Which of the following is an equation for the line that is tangent to the graph of $f(x) = x^3 - 9x + 5$ at $x = 2$?

(a) $y = 3x - 17$
(b) $y = 3x - 11$
(c) $y = 3x - 8$
(d) $y = 3x - 5$
(e) $y = 3x + 5$

2. Which of the following is the derivative of $g(x) = 3\sqrt{5x + 2}$?

(a) $\dfrac{15}{2\sqrt{5x + 2}}$
(b) $\dfrac{3}{2\sqrt{5x + 2}}$
(c) $\dfrac{5}{6\sqrt{5x + 2}}$
(d) $\dfrac{30}{\sqrt{5x + 2}}$
(e) $\dfrac{15}{\sqrt{5x + 2}}$

3. Which of these is the limit $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{x^2 - 5x + 6}$?

(a) 1
(b) $\dfrac{4}{5}$
(c) -4
(d) 0
(e) Does not exist

4. Which of the following is the derivative of $f(x) = x^3 \sin(4x + 1)$?

(a) $-3x^2 \cos(4x + 1)$
(b) $12x^2 \cos(4x + 1)$
(c) $3x^2 \sin(4x + 1) + x^3 \cos(4x + 1)$
(d) $3x^2 \sin(4x + 1) - x^3 \cos(4x + 1)$
(e) $3x^2 \sin(4x + 1) + 4x^3 \cos(4x + 1)$

5. Which of the following is the derivative of $f(x) = \ln(x^2 + 4)$?

(a) $2x$

(b) $\dfrac{1}{2x}$

(c) $\dfrac{2x}{x^2 + 4}$

(d) $\dfrac{1}{x^2 + 4}$

(e) $\dfrac{-1}{(x^2 + 4)^2}$

6. Which of these is the derivative of $h(x) = \tan(5x + 4)$?

(a) $\sec(5x + 4)$

(b) $5\sec^2(5x + 4)$

(c) $5\sec(5x + 4)\tan(5x + 4)$

(d) $\sec(5x + 4)\tan(5x + 4)$

(e) $5\cot(5x + 4)$

7. The derivative of the function $f(x)$ is $f'(x) = 6x^2 - 10$. If $f(1) = 15$, which of the following is the formula for $f(x)$?

(a) $12x^3 - 10x$

(b) $12x^3 - 10x + 15$

(c) $2x^3 + 13$

(d) $2x^3 - 10x + 15$

(e) $2x^3 - 10x + 23$

8. A particle is traveling around the circle $x^2 + y^2 = 25$ where x and y are measured in millimeters. At the instant the particle is at the point $(-4, 3)$, $dy/dt = 16$ mm/sec. Find dx/dt at this time.

(a) 12 mm/sec

(b) 9 mm/sec

(c) 24 mm/sec

(d) −12 mm/sec

(e) −16 mm/sec

13. Which of the following is the derivative of $g(x) = \cos(x^3 + 4)$?

(a) $-3x^2 \sin(x^3 + 4)$
(b) $3x^2 \sin(x^3 + 4)$
(c) $\sin(x^3 + 4)$
(d) $-\sin(x^3 + 4)$
(e) $\sin(3x^2)$

14. Let $f(x) = x^3 + 6x + 2$. Use Newton's method to approximate where $f(x)$ has a zero. Start with $x_1 = 1$ and calculate x_2 and x_3.

(a) $x_2 = 2$ and $x_3 = \frac{20}{9}$
(b) $x_2 = 2$ and $x_3 = \frac{7}{9}$
(c) $x_2 = 2$ and $x_3 = \frac{2}{3}$
(d) $x_2 = 0$ and $x_3 = \frac{1}{3}$
(e) $x_2 = 0$ and $x_3 = \frac{-1}{3}$

9. Which of the following is the limit $\lim_{x\to 0} \frac{\sin(3x)}{2x}$?

(a) 0

(b) $\frac{2}{3}$

(c) 1

(d) $\frac{3}{2}$

(e) Does not exist

10. Which of the following is the limit $\lim_{x\to \infty} \frac{x^2 + 5e^x}{2e^x}$?

(a) 0

(b) $\frac{5}{2}$

(c) $\frac{7}{2}$

(d) $+\infty$

(e) Does not exist

11. Which of the following is the derivative of $f(x) = x^2e^{5x+1}$?

(a) $10xe^{5x+1}$

(b) $10xe^{5x}$

(c) $2xe^{5x+1} + 5x^2e^{5x}$

(d) $2xe^{5x+1} + x^2e^{5x+1}$

(e) $2xe^{5x+1} + 5x^2e^{5x+1}$

12. Which of the following is the second derivative of $f(x) = (x^3 + 4)^5$?

(a) $20(x^3 + 4)^3$

(b) $360x^3(x^3 + 4)^5$

(c) $20(6x)^3$

(d) $30x(x^3 + 4)^4 + 180x^4(x^3 + 4)^3$

(e) $30x(x^3 + 4)^4 + 60x^2(x^3 + 4)^3$

MATH 1241 – CALCULUS I

SPRING 2016

COMMON FINAL EXAMINATION

UNC CHARLOTTE
Department of Mathematics and Statistics

Last Name: _____ First Name: _____
(Please PRINT) (Please PRINT)

Student ID #: _____ Instructor: _____ Section: _____

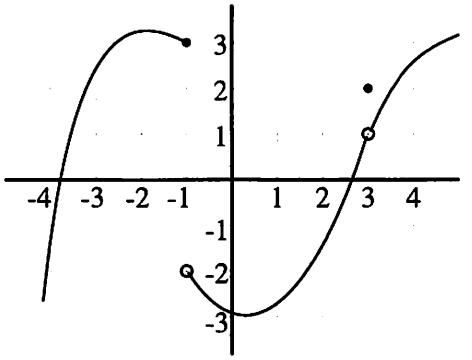
PART II

- Part II consists of 11 multiple choice problems. After your exam proctor announces that calculator may be used, you may use your calculator on this part of the exam. (Texas Instruments 83, 84, 89 or equivalent models of other brands are allowed. **TI Inspire**, TI 92 or equivalent calculators are NOT allowed at all on this exam.)
- You must use a pencil with soft black lead (#2 or HB) to indicate your answers on the Opscan sheets.
- For each question, choose the response which best fits the question.
- If you wish to change an answer, make sure that you completely erase your old answer and any extraneous marks.
- There is no penalty for guessing.
- If you mark more than one answer to a question, that question will be marked as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the Opscan sheets.
- Make sure that your name appears on the Opscan sheets and that you fill in the circles corresponding to your name in the format Last, First.
- At the end of the exam you must hand in all test material including the test booklets, Opscan sheets and scratch paper.

MATH 1241 FINAL EXAM SPRING 2016
Part II, Calculators Allowed

1. Determine the values of A and B (if they exist) using the graph of $f(x)$

$\lim_{x\to -1^+} f(x) = A \quad \lim_{x\to 3} f(x) = B$



(a) $A = 3, \quad B = 2$
(b) $A = 3, \quad B = 1$
(c) $A = -2, \quad B = 2$
(d) $A = -2, \quad B = 1$
(e) A does not exist, $\quad B = 2$

2. Find an equation for the line that is tangent to the curve $14x = y^3 - 2y + 38$ at $(3, 2)$.
(a) $y = 1.4x - 2.2$
(b) $y = 1.4x + 6.4$
(c) $y = -x + 5$
(d) $y = -x + 2$
(e) $y = x - 1$

3. Find the x and y coordinate of each point on the graph of $y = x^2 + 5x + 8$ where the slope of the tangent line is 3.
(a) $(-1, 4)$
(b) $(1, 14)$
(c) $(-1, 0)$
(d) $(1, 0)$
(e) $(1.5, 17.75)$

4. A cube is measured to have sides of length 8 cm with a possible error of ± 0.2 cm. Use differentials to estimate the maximum error in calculating the volume.
(a) $\pm 38.4\,\text{cm}^3$
(b) $\pm 7.68\,\text{cm}^3$
(c) $\pm 4.8\,\text{cm}^3$
(d) $\pm 4.096\,\text{cm}^3$
(e) $\pm 0.008\,\text{cm}^3$

5. The graph at the right is the graph of the **derivative** $f'(x)$ of the function $f(x)$. Determine the x-coordinate of each local maximum and each local minimum of the **function** $f(x)$.

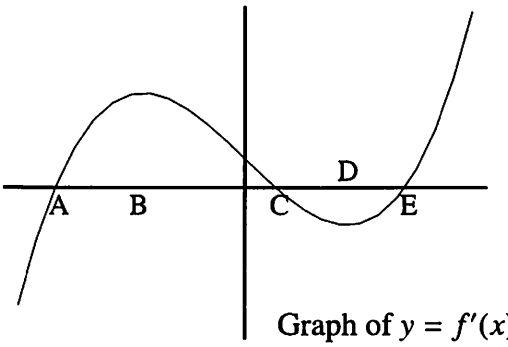
(a) Local max at $x = B$,
local min at $x = D$

(b) Local max at $x = A$,
local min at $x = C$

(c) Local max at $x = A$ and $x = E$
local min at $x = C$

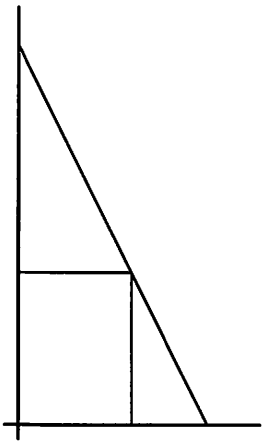
(d) Local max at $x = C$
local min at $x = A$ and $x = E$

(e) Local max at $x = C$
local min at $x = E$



Graph of $y = f'(x)$

6. The origin $(0, 0)$ is one vertex of a rectangle, another is on the positive x-axis, a third on the positive y-axis and the fourth is on the line $y = 5 - 2x$, as in diagram below. What is the largest possible area of such a rectangle?



(a) 2.75
(b) 3.125
(c) 3.25
(d) 3.75
(e) 4.25

7. Determine the values of b and c so that the function $f(x) = \begin{cases} 3x + b & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x < 2 \\ cx + 10 & \text{if } 2 \le x \end{cases}$ is continuous.

(a) $b = -2$ & $c = 9$
(b) $b = 10$ & $c = 3$
(c) $b = 4$ & $c = -3$
(d) $b = 2$ & $c = -3$
(e) Impossible to make continuous

Use the following table of values for problems 8, 9 and 10.

x	$f(x)$	$f'(x)$		$g(x)$	$g'(x)$
0	2	-7		1	10
1	4	10		5	-7
2	0	-5		4	9
3	5	8		2	14
4	3	100		3	-8
5	1	4		0	200

For example, for $x = 5$: $f(5) = 1$, $f'(5) = 4$, $g(5) = 0$ and $g'(5) = 200$

8. Calculate the value of $r'(1)$ for $r(x) = f(x)g(x)$.

(a) -70
(b) -50
(c) 22
(d) 48
(e) 64

9. Calculate the value of $h'(2)$ for $h(x) = g(f(x))$.

(a) 10
(b) 5
(c) 1
(d) 0
(e) -50

10. Calculate the value of $k'(4)$ for $k(x) = x^2g(x)$.

(a) -128
(b) -104
(c) -64
(d) 24
(e) 128

11. The derivative of the function $f(x) = x^3 - 60x^2 + 10,000$ is $f'(x) = 3x^2 - 120x = 3x(x-40)$. Determine the absolute maximum value and the absolute minimum value of $f(x)$ on the interval $[-15, 20]$.

(a) Absolute maximum is 10,000, absolute minimum is -22,000

(b) Absolute maximum is -6000, absolute minimum is -22,000

(c) Absolute maximum is -6000, absolute minimum is -6875

(d) Absolute maximum is 10,000, absolute minimum is -6875

(e) Absolute maximum is 2474, absolute minimum is -2400

MATH 1241 – CALCULUS I

SPRING 2016

COMMON FINAL EXAMINATION

UNC CHARLOTTE
Department of Mathematics and Statistics

Last Name: _____ First Name: _____
(Please PRINT) (Please PRINT)

Student ID #: _____ Instructor: _____ Section: _____

For Grading Use Only:

Problem	1	2	3	4	5
Grade					
Out of					

Free Response Score

PART III

- Part III consists of 5 free response problems. After your exam proctor announces that calculator may be used, you may use your calculator on this part of the exam. (Texas Instruments 83, 84, 89 or equivalent models of other brands are allowed. **TI Inspire**, TI 92 or equivalent calculators are NOT allowed at all on this exam.)
- Please show all of your work on the problem sheet provided. Work that is done on scratch paper or any other sheets will not be graded.
- You may use your calculator to check your answers, but complete justification must be shown for each problem. This includes all graphs, calculations and references to supporting theorems.
- Make sure that your name appears on each page of the test booklet.
- At the end of the exam you must hand in all test material including the test booklets, Opscan sheets and scratch paper.

MATH 1241 FINAL EXAM SPRING 2016
Part III, Calculators Allowed

You must show your work, otherwise substantial point reductions may occur.

1. The following information is known about a function f.
   (a) $f(x)$, $f'(x)$ and $f''(x)$ are continuous functions defined for all values of x.
   (b) $f'(x) = 0$ when $x = -2$, $x = 0$ and $x = 5$, and $f''(x) = 0$ when $x = -4$, $x = -3$, $x = -2$, $x = -1$ and $x = 3$.
   (c) $f'(x) > 0$ when $-2 < x < 0$ and $5 < x$, and $f'(x) < 0$ when $x < -2$ and $0 < x < 5$.
   (d) $f''(x) > 0$ when $x < -4$, $-3 < x < -1$ and $3 < x$, and $f''(x) < 0$ when $-4 < x < -3$ and $-1 < x < 3$.

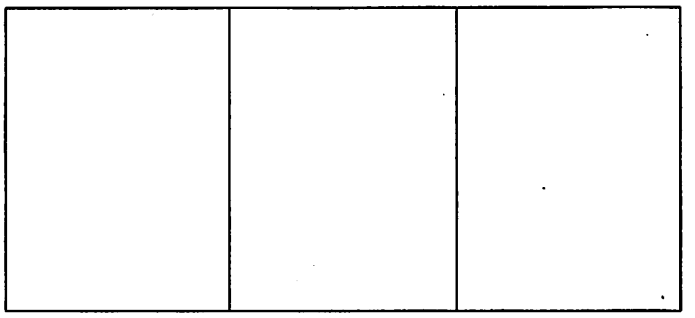
(A) On what intervals, if any, is the function $f(x)$ increasing?

(B) For what values of x, if any, does $f(x)$ have a local maximum?

(C) On what intervals, if any, is the graph of function $f(x)$ concave down?

(D) For what values of x, if any, does $f(x)$ have an inflection point?

2. A very large rectangular area is to fenced in and divided into three smaller fenced in areas as in the illustration below. If there is 4000 feet of fencing available, what are the dimensions of the largest total area that can be fenced.



3. A rocket is launched and its height for $t \geq 0$ is given by $h(t) = 10t^3 + 40t$ where t is measured in seconds and $h(t)$ is measured in meters.

(A) Determine the height at $t = 5$ and at $t = 15$.

(B) Determine the average velocity from $t = 5$ to $t = 15$

(C) Determine the instantaneous velocity at $t = 10$.

4. A spherical balloon is being inflated at the constant rate of $12\,\text{in}^3$ per second.

(A) What is the volume of the balloon when the radius is 10 in? (Volume is $V = (4/3)\pi r^3$)

(B) At what rate is the radius r increasing when $r = 10$ in?

(C) At what rate is the surface area changing when $r = 10$? (Surface area is $S = 4\pi r^2$)

5. Let $f(x) = x^3 - 5x^2 + 12$.

(A) Find the slope of the line through the points $(1, f(1))$ and $(4, f(4))$.

(B) Verify the conclusion of the Mean Value Theorem by finding a number $1 < c < 4$ where the slope of line tangent to $f(x)$ at $x = c$ is the same as the slope of the line through $(1, f(1))$ and $(4, f(4))$.