MATH 1242 SPRING 2014
COMMON FINAL EXAMINATION
PART I

Name:________________________ Instructor:____________________

Student ID #:____________________ Section/Time:__________________

This exam is divided into three parts. **Calculators are not allowed on Part I.** You have two hours and 45 minutes for the entire test, but you have only one hour to finish Part I. You may start working on the other two parts of the test whenever you are done with Part I, but you cannot use your calculator until after 9:00 am. Your instructor will announce that the calculators are allowed to work on Part II and Part III.

These pages contain Part I which consists of 15 multiple choice questions. These questions must be answered without the use of a calculator.

- You must use a pencil with a soft black lead (#2 or HB) to enter your answers on the answer sheet.
- For each question choose the response which best fits the question.
- If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.
- There is no penalty for guessing.
- If you mark more than one answer to a question, that question will be scored as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.
- **Make sure that your name appears on the answer sheet and that you fill in the circles corresponding to your name.**

At the end of the examination you MUST hand in all test materials including test booklets, answer sheet and scratch paper.

Part I, No Calculators Allowed

1. Evaluate the integral: $\int (6x^2 - 10x + 7)\,dx$.

(a) $18x^3 - 20x^2 + 7x + C$

(b) $18x^3 - 20x^2 + 7 + C$

(c) $6x^3 - 10x^2 + 7x + C$

(d) $2x^3 - 5x^2 + 7 + C$

(e) $2x^3 - 5x^2 + 7x + C$

2. Evaluate the integral: $\int \cos(2x)\,dx$.

(a) $0.5 \sin(2x) + C$

(b) $-0.5 \sin(2x) + C$

(c) $-\sin(2x) + C$

(d) $-\sin(x^2) + C$

(e) $\sin(x^2) + C$

3. Which of the following is true about the sequence $\{a_n\}$ where $a_n = \dfrac{n+1}{3n+4}$?

(a) $\lim_{n\to\infty} a_n = +\infty$

(b) $\lim_{n\to\infty} a_n = \dfrac{1}{3}$

(c) $\lim_{n\to\infty} a_n = \dfrac{1}{4}$

(d) $\lim_{n\to\infty} a_n = \dfrac{2}{7}$

(e) There is no limit

4. Evaluate the integral $\int e^{2x+1} dx$.

(a) $e^{2x+1} + C$

(b) $2\,e^{2x+1} + C$

(c) $(1/2)\,e^{2x+1} + C$

(d) $e^{x^2+1} + C$

(e) $x\,e^{2x+1} - e^{2x+1} + C$

5. Which of the following is true about the series $\sum_{n=0}^{\infty} \frac{3}{2^n}$?

(a) It diverges because $\lim_{k\to\infty} \frac{3}{2^k} = 0$

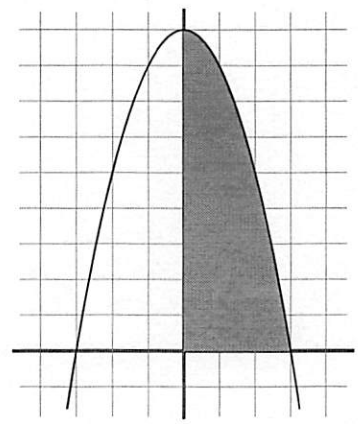
(b) It diverges because $\lim_{k\to\infty} \sum_{n=1}^{k} \frac{3}{2^n} = +\infty$

(c) It converges to 2.

(d) It converges to 3.

(e) It converges to 6.

6. The shaded region in the picture is bounded above (and right) by the graph of $f(x) = 9 - x^2$, below by the x-axis and on the left by the y-axis. What is the area of this region?



(a) 9

(b) 16

(c) 18

(d) 20

(e) 21

7. Evaluate the integral: $\int_0^2 40(1-x)^3\,dx$.

(a) -640

(b) -160

(c) 0

(d) 160

(e) 640

8. Evaluate the integral $\int 8x\cos(2x)\,dx$.

(a) $2x^2\sin(2x) + C$

(b) $-2x^2\sin(2x) + C$

(c) $8x^2\sin(2x) + C$

(d) $4x^2\cos(2x) + 4x\sin(2x) + C$

(e) $4x\sin(2x) + 2\cos(2x) + C$

9. Evaluate the integral $\int_1^\infty \frac{6}{x^2}\,dx$.

(a) 0

(b) 2

(c) 3

(d) 6

(e) Diverges to $+\infty$

10. What is the coefficient of x^3 in the Maclauren series expansion of $\sin(2x)$?

(a) $-4/3$

(b) $-1/3$

(c) $-1/6$

(d) $1/6$

(e) $1/3$

11. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$. Which of the following tests results in a successful determination of the convergence or divergence of this series?

(a) Converges by the integral test

(b) Diverges by the ratio test

(c) Converges by the ratio test

(d) Converges by comparison with $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$

(e) Converges by the alternating series test

12. From a table of integrals: $\int \frac{1}{u\sqrt{u^2-1}}\,du = \operatorname{arcsec}(u)$. Use this to evaluate the integral $\int \frac{1}{x\sqrt{4x^2-1}}\,dx$.

(a) $(1/4)\operatorname{arcsec}(2x) + C$

(b) $(1/2)\operatorname{arcsec}(2x) + C$

(c) $\operatorname{arcsec}(2x) + C$

(d) $2\operatorname{arcsec}(2x) + C$

(e) $4\operatorname{arcsec}(2x) + C$

13. Evaluate the integral $\int \frac{5}{x^2-4}\,dx$.

(a) $\frac{-5}{x-2} + C$

(b) $5\arctan(x) + C$

(c) $5\arctan(x/2) + C$

(d) $5\ln|x-2| - 5\ln|x+2| + C$

(e) $(5/4)\ln|x-2| - (5/4)\ln|x+2| + C$

14. Suppose $f'(x) = 6x^2 - 5$ and $f(1) = 12$. What is $f(0)$?

(a) 0

(b) 11

(c) 15

(d) 19

(e) Impossible to determine.

15. Which of the following is the derivative of $f(x)$ where $f(x) = \int_0^{2x} \sin(t^2+1)\,dt$?

(a) $2\sin(4x^2+1)$

(b) $\sin(x^2+1)$

(c) $8x\sin(4x^2+1)$

(d) $4x\cos(x^2+1)$

(e) $-2x\cos(x^2+1)$

MATH 1242 SPRING 2014
COMMON FINAL EXAMINATION
PART II

Name:________________________ Instructor:____________________

Student ID #:____________________ Section/Time:__________________

These pages contain Part II which consists of 10 multiple choice questions. After your instructor announces that calculators are OK, you are allowed to use a calculator to this part of the exam.

- You must use a pencil with a soft black lead (#2 or HB) to enter your answers on the answer sheet.
- For each question choose the response which best fits the question.
- If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.
- There is no penalty for guessing.
- If you mark more than one answer to a question, that question will be scored as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.
- **Make sure that your name appears on the answer sheet and that you fill in the circles corresponding to your name.**

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MATH 1242 **FINAL EXAM** **SPRING 2014**

Part II, Calculators Allowed

1. Which of the following integrals gives the length of the curve $y = x^3 - 2x$ on the interval $[1, 5]$?

(a) $\int_1^5 \sqrt{x^6 - 4x^4 + 4x^2 + 1}\, dx$

(b) $\int_1^5 \sqrt{x^6 - 4x^4 + 4x^2 - 1}\, dx$

(c) $\int_1^5 \sqrt{3x^2 - 1}\, dx$

(d) $\int_1^5 \sqrt{9x^4 - 12x^2 + 5}\, dx$

(e) $\int_1^5 \sqrt{9x^4 - 12x^2 + 3}\, dx$

2. Consider the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$. Which of the following tests results in a successful determination of the convergence or divergence of this series?

(a) Diverges by the integral test

(b) Converges by the integral test

(c) Converges by the ratio test

(d) Diverges by the comparison test using $\sum_{n=1}^{\infty} \frac{1}{2n}$

(e) Converges by the comparison test using $\sum_{n=1}^{\infty} \frac{1}{2n}$

3. Which of the following definite integrals gives the x-coordinate of the centroid of the region in the first quadrant that is bounded above by the graph of $f(x) = 8 - x^3$ and below by the x-axis?

(a) $\frac{1}{12}\int_0^2 (8x - x^4)\, dx$

(b) $\int_0^2 (8x - x^4)\, dx$

(c) $\frac{1}{12}\int_0^8 y\sqrt[3]{8-y}\, dy$

(d) $\frac{1}{6}\int_0^8 y\sqrt[3]{8-y}\, dy$

(e) $\int_0^8 y\sqrt[3]{8-y}\, dy$

4. Which of the following is true about the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n}$?

(a) It diverges because $\lim_{n\to\infty} \frac{(-1)^n}{2n} = 0$

(b) It diverges because $\lim_{k\to\infty} \sum_{n=1}^{n=k} \frac{1}{2n} = +\infty$

(c) It converges to a negative number

(d) It converges to a positive number

(e) It converges to 0

5. Which of the following gives the best estimate to five decimal places (so nearest 0.00001) of the absolute value of the error E in using 10 equally wide trapezoids to approximate the area under the graph of $f(x) = 30 - x^2$ on the interval $[1, 5]$? [In general the error is $|E| = \frac{(b-a)^3\,|f''(c)|}{12n^2}$ where n is the number of trapezoids and c is some number in the interval (a, b) (that may change depending on n).]

(a) 0.01067

(b) 0.01433

(c) 0.10667

(d) 0.20833

(e) 0.36000

6. The only information known about the function $f(x)$ is that provided by the table of values given below. Estimate the value of the definite integral $\int_0^2 f(x)\,dx$ using a Riemann sum where the interval $[0, 2]$ is divided into four subintervals of equal width and the midpoint of each subinterval is used as the sample point. [The same as using the "Midpoint Rule" to estimate the integral.]

x	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
$f(x)$	1	1.25	2.5	2.75	1.5	1.4	3.2	4.5	5

(a) 4.95

(b) 5.1

(c) 9.9

(d) 10.2

(e) 20.4

7. An object on the x-axis is subject to a force of x^2 pounds (with x measured in feet). How much work is done in moving the object from $x = 1$ to $x = 10$?

(a) 33 ft-lb

(b) 99 ft-lb

(c) 333 ft-lb

(d) 999 ft-lb

(e) 2499.75 ft-lb

8. What is the average value of the function $f(x) = 3x^2 - 5x + 8$ on the interval $[-1, 5]$?

(a) 7

(b) 19

(c) $\frac{37}{3}$

(d) 37

(e) 114

9. A particle is moving along the x-axis at a rate of $v(t) = 6t^2 + 1$ cm/s at time t. How far does the particle travel from $t = 1$ to $t = 4$?

(a) 43 cm

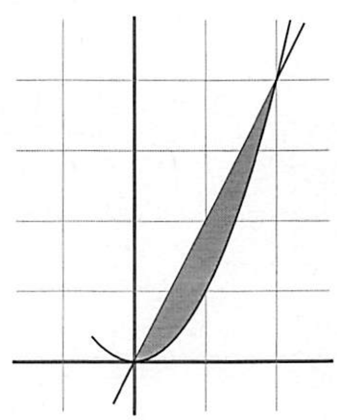
(b) 44 cm

(c) 90 cm

(d) 129 cm

(e) 132 cm

10. Which of the definite integrals will give the volume of the solid obtained by rotating the area bounded by the graphs $f(x) = 2x$ and $g(x) = x^2$ (the shaded region in the picture) around the x-axis?



(a) $\int_0^2 2\pi x\,(2x - x^2)\,dx$

(b) $\int_0^2 2\pi x\,(2x + x^2)\,dx$

(c) $\int_0^2 \pi\,(2x - x^2)^2\,dx$

(d) $\int_0^2 \pi\,(4x^2 - x^4)\,dx$

(e) $\int_0^2 2\pi\,(4x^2 - x^4)\,dx$

MATH 1242
COMMON FINAL EXAMINATION
FREE RESPONSE SECTION
SPRING 2014

This exam is divided into three parts. These pages contain Part III which consists of 5 free response questions.

Please show all of your work on the problem sheet provided. We will not grade loose papers.

- If you are basing your answer on a graph on your calculator, sketch a picture of your graph on your sheet and be sure to label your window.
- **Make sure that your name appears on each page.**

At the end of the examination you MUST hand in all remaining test materials including test booklets, answer sheet, and scratch paper.

PROBLEM	1	2	3	4	5
GRADE					

FREE RESPONSE SCORE: __________

Name: ________________________ Student ID No: ____________________

Instructor: ____________________ Section No: ____________________

MATH 1242 **FINAL EXAM** **SPRING 2014**

Part III, Calculators Allowed

Note: Even though calculators are allowed, you must show your work to receive credit.

1. The Taylor series expansion for $\dfrac{1}{x+1}$ about $x = 0$ is $1 - x + x^2 - x^3 + \cdots$.

(a) Use the Taylor series expansion for $\dfrac{1}{x+1}$ to find the first four nonzero terms of the Taylor series expansion for $\dfrac{1}{x^3+1}$ about $x = 0$.

(b) Use the Taylor series for $\dfrac{1}{x^3+1}$ from part (a) to evaluate the integral $\displaystyle\int_0^x \frac{1}{t^3+1}\,dt$. Give the answer as a Taylor series about $x = 0$. As in part (a), give at least the first four nonzero terms.

(c) Use the first three nonzero terms of the Taylor series expansion of $\displaystyle\int_0^x \frac{1}{t^3+1}\,dt$ from part (b) to approximate the definite integral $\displaystyle\int_0^{1/2} \frac{1}{t^3+1}\,dt$. Round your answer to five decimal places (so to the nearest 0.00001).

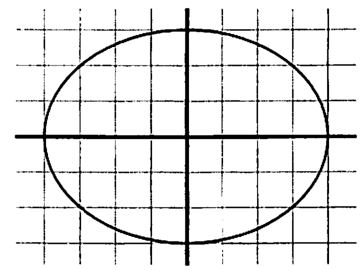
2. (a) Determine the partial fraction decomposition of $f(x) = \dfrac{3x^2 + 5}{x^2 + x}$. [First you will need to do long division.]

(b) Use the decomposition from part (a) to evaluate the integral $\displaystyle\int \frac{3x^2 + 5}{x^2 + x}\,dx$. [You must show your work.]

3. Find the interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{2^n(n + 4)}$. [Don't forget to check the convergence at the endpoints. Also, in determining the convergence behavior at the endpoints, include what test was used.]

4. The ellipse given by the equation $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is rotated around the x-axis and also around the y-axis.

(a) Use the "disk" method to determine the volume of the solid that results from rotating the ellipse around the x-axis.



(b) Use the method of "cylindrical shells" to determine the volume of the solid that results from rotating the ellipse around the y-axis.

5. Evaluate the following integrals. [Remember, you must show all of the steps involved in doing the integration to receive credit.]

(a) $\int x^2 e^{5x}\,dx$

(b) $\int \sin(3x)\cos(3x)\,dx$