MATH 1242 – CALCULUS II

SPRING 2015

COMMON FINAL EXAMINATION

UNC CHARLOTTE
Department of Mathematics and Statistics

Last Name: _____ First Name: _____
(Please PRINT) (Please PRINT)

Student ID #: _____ Section: _____

PART II

- Part II consists of 10 multiple choice problems. After your exam proctor announces that calculator may be used, you may use your calculator on this part of the exam. (Texas Instruments 83, 84, 89 or equivalent models of other brands are allowed. **TI Inspire**, TI 92 or equivalent calculators are NOT allowed at all on this exam.)
- You must use a pencil with soft black lead (#2 or HB) to indicate your answers on the Opscan sheets.
- For each question, choose the response which best fits the question.
- If you wish to change an answer, make sure that you completely erase your old answer and any extraneous marks.
- There is no penalty for guessing.
- If you mark more than one answer to a question, that question will be marked as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the Opscan sheets.
- Make sure that your name appears on the Opscan sheets and that you fill in the circles corresponding to your name in the format Last, First.
- At the end of the exam you must hand in all test material including the test booklets, Opscan sheets and scratch paper.

Part II (MULTIPLE CHOICE, CALCULATORS ALLOWED).

1. Consider the following table of values for a function f. (For example, $f(1/2) = 6$.) Use the table to compute the Riemann sum for f on the interval $[0, 2]$, using four subintervals of equal width and taking the sample points to be the right endpoints of the subintervals.

t	0	1/2	1	3/2	2
$f(t)$	1	6	4	-1	2

(a) 6

(b) 11/2

(c) 22

(d) 11

(e) 8

2. Consider the function f of problem 1. Which of the following is the approximation obtained when the midpoint rule with *two* subintervals of equal width is used to approximate $\int_0^2 f(x)\,dx$?

(a) 5

(b) 5/2

(c) 6

(d) 12

(e) 10

3. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n^2}$.

(a) $[-1, 1]$

(b) $(-1, 1)$

(c) $[2, 4)$

(d) $[2, 4]$

(e) $(2, 4]$.

4. A spring has a natural length of 0.5 m. If a force of 100 N is required to keep the spring stretched to a length of 1.0 m, how much work is done in stretching the spring from its natural length to a length of 2.5 m?

(a) 100 J

(b) 200 J

(c) 300 J

(d) 400 J

(e) 500 J

5. Let $f(x) = x^4 - 2x^3 + 7x^2 - x - 5$. Use a Taylor expansion to find a_3 if we rewrite $f(x)$ as $f(x) = a_0 + a_1(x-2) + a_2(x-2)^2 + a_3(x-2)^3 + a_4(x-2)^4$.

(a) 1/6

(b) 6

(c) 36

(d) $\pi/6$

(e) 4

6. On a faraway planet (in a faraway galaxy), a creature is walking in a straight line. If it has walked a distance of $D(t) = \int_0^t \sqrt{u^2+7}\,du$ km in t sec, at what rate is it walking after 3 sec ($t = 3$)?

(a) 0 km/sec

(b) 3/8 km/sec

(c) 4 km/sec

(d) 1 km/sec

(e) 2 km/sec

7. Which of the following is the best approximation to the length of the curve $y = x^4$, $0 \leq x \leq 1$? (Use your calculator to evaluate the integral involved.)

(a) 1.6

(b) 1.4

(c) 1.2

(d) 1.0

(e) 0.8

8. Consider the region in the plane bounded above by the curve $y = \sqrt{x^2 + 1}$ and below by the x-axis for $0 \leq x \leq 1$. Find the volume of the solid obtained by rotating the region about the x-axis.

(a) 2.000

(b) 6.283

(c) 1.478

(d) 3.606

(e) 4.189

9. In the partial fractions expansion $\dfrac{5x+1}{x^2-1} = \dfrac{A}{x-1} + \dfrac{B}{x+1}$, find A.

(a) 3/2

(b) 1

(c) 3

(d) 2

(e) -3

10. A particle is moves along the x-axis. The velocity of the particle at time t is $3\sqrt{t}$ cm/s. What is the average velocity of the particle for $0 \leq t \leq 2$?

(a) 0.53 cm/s

(b) 1.06 cm/s

(c) 22.12 cm/s

(d) 2.83 cm/s

(e) 5.66 cm/s

11. How far does the particle travel in problem 10 (for $0 \leq t \leq 2$)?

(a) 5.66 cm

(b) 4.24 cm

(c) 2.83 cm

(d) 0.53 cm

(e) 1.06 cm

12. Use the expansion $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$ (valid for $-1 < x < 1$) to obtain the Maclaurin expansion of $\frac{1}{1+x^4}$.

(a) $\frac{1}{1+x^4} = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \cdots$

(b) $\frac{1}{1+x^4} = 1 - x + x^2 - x^3 + \cdots$

(c) $\frac{1}{1+x^4} = 1 + x + x^2 + x^3 \cdots$

(d) $\frac{1}{1+x^4} = 1 - x^4 + x^8 - x^{12} + \cdots$

(e) $\frac{1}{1+x^4} = 1 + x^4 + x^8 + x^{12} + \cdots$

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This exam is divided into three parts. Calculators are not allowed on Part I. You have 3 hours for the entire exam, but you have only one hour to finish Part I. You may start working on the other two parts of the test after the first hour, but you cannot use your calculator during this time. You may use your calculator ONLY after your exam proctor has announced that calculators are allowed on Parts II and III. (Texas Instruments 83, 84, 89 or equivalent models of other brands are allowed. **TI Inspire**, TI 92 or equivalent calculators are NOT allowed at all on this exam.)

PART I

- Part I consists of 15 multiple choice problems. These problems must be answered without the use of a calculator.
- You must use a pencil with soft black lead (#2 or HB) to indicate your answers on the Opscan sheets.
- For each question, choose the response which best fits the question.
- If you wish to change an answer, make sure that you completely erase your old answer and any extraneous marks.
- There is no penalty for guessing.
- If you mark more than one answer to a question, that question will be marked as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the Opscan sheets.
- Make sure that your name appears on the Opscan sheets and that you fill in the circles corresponding to your name in the format Last, First.
- At the end of the exam you must hand in all test material including the test booklets, Opscan sheets and scratch paper.

Part I (MULTIPLE CHOICE, NO CALCULATORS).

1. $\int x^5\,dx = ?$

(a) $5x^4 + C$

(b) $\frac{1}{5}x^6 + C$

(c) $\frac{1}{5}x^5 + C$

(d) $\frac{1}{6}x^6 + C$

(e) $\ln(|x^5|) + C$

2. $\int_1^2 \frac{3}{x}\,dx = ?$

(a) 1

(b) $\frac{8}{3}$

(c) $-\frac{3}{2}$

(d) $\ln(2)$

(e) $3\ln(2)$

3. $\int x^3(2x^4+1)^4\,dx = ?$

(a) $\frac{1}{40}(2x^4+1)^5 + C$

(b) $\frac{1}{80}(2x^4+1)^5 + C$

(c) $\frac{1}{40}x^3(2x^4+1)^5 + C$

(d) $\frac{1}{5}(2x^4+1)^4 + C$

(e) $\frac{8}{5}(2x^4+1)^4 + C$

4. $\int x^2 \ln(x)\, dx = ?$

(a) $2x \ln(x) + C$

(b) $\frac{1}{3}x^3 \ln(x) - \frac{1}{9}x^3 + C$

(c) $\frac{1}{3}x^3 \ln(x) - \frac{1}{3}x^3 + C$

(d) $x^3 \ln(x) - \frac{1}{3}x^3 + C$

(e) $\frac{1}{3}x^3 \ln(x) + C$

5. $\int \sin^3(x) \cos(x)\, dx = ?$

(a) $\sin^4(x) + C$

(b) $\sin^4(x) \cos(x) + C$

(c) $\cos^3(x) \sin(x) - 3\sin^2(x) \cos(x) + C$

(d) $\frac{1}{4}\sin^5(x) + C$

(e) $\frac{1}{4}\sin^4(x) + C$

6. $\int_1^2 \frac{2}{t^3}\, dt = ?$

(a) 0

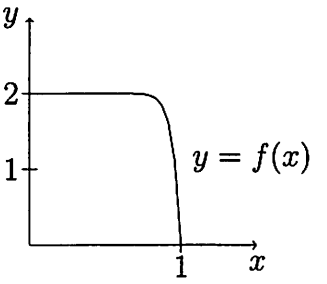
(b) $-\frac{1}{2}$

(c) $-\frac{1}{4}$

(d) $\frac{3}{4}$

(e) $2\ln(8)$

7. Consider the graph of the function f :



Which of the following best approximates $\int_0^1 f(x)\,dx$?

(a) 0

(b) 0.2

(c) 1.9

(d) 1.1

(e) 0.9

8. Find the area of the region bounded above by the by the graph of $y = 4x^3$ and below by the line $y = 4$ for $1 \leq x \leq 2$.

(a) 11

(b) 16

(c) 28

(d) 36

(e) 48

9. The sequence $\{a_n\}$, where $a_n = \dfrac{1}{n^2+1}$,

(a) converges to 0.

(b) converges to 1/2.

(c) converges to 1.

(d) converges to $1/n$.

(e) diverges.

10. The series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

(a) converges to 0.

(b) converges to $1/n$.

(c) converges to $1/2$.

(d) converges to a number greater than $1/2$.

(e) diverges.

11. The series $\sum_{n=1}^{\infty} \frac{3^n}{4^{n-1}}$

(a) converges to 4.

(b) converges to 12.

(c) diverges to ∞.

(d) diverges, but not to ∞.

(e) converges to 0.

12. The improper integral $\int_1^{\infty} e^{-x}\,dx$

(a) converges to e.

(b) converges to e^{-1}.

(c) converges to $-e^{-1}$.

(d) converges to 0.

(e) diverges.

13. If a certain function f satisfies $f'(x) = 3x^2 + 1$ and $f(1) = 4$, then $f(2) = ?$

(a) -8

(b) 13

(c) 10

(d) 2

(e) 12

14. The power series $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$ is the Maclaurin expansion of which of the following?

(a) e^x

(b) $\sin(x^2)$

(c) $\cos(x)$

(d) $\cos(x^2)$

(e) $\sin(x)$

15. Let $f(x) = xe^{x^2+1}$. Find the area under the graph of f and above the x-axis for $0 \leq x \leq 2$.

(a) e

(b) 0

(c) $\frac{1}{2}(e^5 - e)$

(d) $2(e^5 - e)$

(e) $e^5 - e$

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For Grading Use Only:

Problem	1	2	3	4	5
Grade					
Out of					

Free Response Score

PART III

- Part III consists of 5 free response problems. After your exam proctor announces that calculator may be used, you may use your calculator on this part of the exam. (Texas Instruments 83, 84, 89 or equivalent models of other brands are allowed. **TI Inspire**, TI 92 or equivalent calculators are NOT allowed at all on this exam.)
- Please show all of your work on the problem sheet provided. Work that is done on scratch paper or any other sheets will not be graded.
- You may use your calculator to check your answers, but complete justification must be shown for each problem. This includes all graphs, calculations and references to supporting theorems.
- Make sure that your name appears on each page of the test booklet.
- At the end of the exam you must hand in all test material including the test booklets, Opscan sheets and scratch paper.

Part III (FREE RESPONSE, CALCULATORS ALLOWED).

Note: Even though calculators are allowed, you must show your work in order to receive credit.

1. For each infinite series below, use an appropriate test to determine whether the series coverges or diverges.

(a) $\sum_{n=1}^{\infty} \frac{n^4}{3^n}$.

(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3n+1}$.

2. Consider the region in the first quadrant bounded above by the curve $y = e^x$ and below by the line $y = 1$ for $0 \leq x \leq 1$. Set up, but do not evaluate, a definite integral that gives the volume of the solid obtained by rotating the region about the y-axis.

3. A tank in the shape of a right-circular cone sits with its base at ground level and its vertex 4 m below ground level. The base has a diameter of 4 m. The tank is full of water. Set up, but do not evaluate, a definite integral that gives the work done in pumping all of the water out of the tank at ground level. (The density of water is 1000 kg/m^3, and the gravitational constant is 9.8 .)

4. (a) Use the expansion $e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \cdots$ (valid for all real numbers x) to obtain an expansion for $\int e^{-x^2}\,dx$.

(b) Use the first three terms of your answer in part (a) to approximate $\int_0^{0.5} e^{-x^2}\,dx$.

(c) Use the Alternating Series Estimation Theorem to find an upper bound for the error in your approximation in part (b).

5. Let R denote the region in the first quadrant bounded above by the line $y = 1$ and below by the curve $y = x^3$, $0 \leq x \leq 1$. Find the centroid of the region R. You may use your calculator to evaluate any definite integrals involved.