MATH 6203-8203: Stochastic Calculus for Finance I
Syllabus: Spring 2019

Text:

- *Stochastic Calculus for Finance I(The Binomial Asset Pricing Model)* by Steve Shreve.
- *Stochastic Calculus for Finance II (Continuous-Time Models)* by Steve Shreve.

Binomial asset pricing model: one-period two state model, actual probability, risk neutral probability, arbitrage, arbitrage free or risk-neutral pricing formula, delta-hedging formula.

Review of General Probability Theory: Sigma-algebra(or sigma field), Axioms of probability measure, Properties of probability measures, Discrete probability space, Uncountable sample space, Borel sets, Borel sigma algebra.

Random Variables and Distribution: Random variables, Distribution measure of a random variable, Cumulative distribution function, Properties of cumulative distribution function, Discrete random variable, Probability mass function, Some examples of discrete random variables, Continuous random variables, Probability density function, Some properties of probability density function, Some examples of continuous random variables with their probability density function and cumulative distribution function.

Expectations: Expected value for when the sample space is finite or countably infinite. Expected value of a discrete random variable. Expected value of a continuous random variable. Computation of Expectation. Moment Generating Function. Convergence of random Variables: Almost Sure Equal. Almost Sure Convergence and Point-wise Convergence, Convergence in distribution and Mean-square convergence. Monotone Convergence and Dominated Convergence.

Change of Measure: Change of measure and Radon-Nikodym derivative process for finite sample space. Change of Measure for uncountable infinite sample space. Equivalent probability measure, Radon-Nikodym derivative of one measure with respect to another measure, Change of measure for a normal random variable, Radon-Nikodym Thoerem.

Information and Conditioning: Filtration, Sigma algebra generated by collection of sets, Sigma algebra generated by a random variable, Independence of events, Independence of sigma algebras. Independence of random variables. Simple way of verifying random variables are independent. Covariance and Correlation. Conditional probability. Radon-Nikodym Theorem. General conditional expectation: Expectation conditioned on a sigma algebra, Properties of Conditional expectation, Martingale property for discrete processes, Martingale Property for Continuous process, Sub-martingale, Super-martingale, Markov processes.

Random walk: Symmetric random walk , Increments of symmetric random walk, martingale property of symmetric random walk. Quadratic Variation of symmetric random walk. Limiting distribution of the scaled random walk.

Brownian motion: Definition of Brownian motion. Filtration for Brownian motion. Martingale property of Brownian motion, Exponential martingale, Markov property of Brownian motion, Quadratic variation: First order variation, Quadratic variation of Brownian motion. Volatility of Geometric Brownian motion.

Stochastic Calculus: Itô Integral for simple integrands, Itô Integral for general integrands, Some properties of Itô integrals, Riemann Integral of Brownian motion, Itô-Doeblin Formula for Brownian motion. Solved some examples. Itô process, Quadratic variation of Itô processes, Integration with respect to Itô processes. Itô-Doeblin formula for Itô processes. Integration by part formula, Solved examples. Itô Integral of deterministic integrand, Generalize Geometric Brownian motion.Some applications to mean-reverting models: Vasicek model and CIR model. 2-Dimensional Itô-Doeblin formula, Product rule and quotient rule formula.

Black-Sholes-Merton Model: Risk-free asset process, Evolution of portfolio value process, Discounted stock price, Discounted portfolio value, Evolution of option value, Derivation of the celebrated Black-Scholes-Merton partial differential equation. Second-oder partial differential equation: Homogeneous heat equation, Derivation of the solution of the Black-Scholes-Merton partial differential equation, The Greeks, Put-Call parity.

Risk-Neutral Pricing: Review of change of measure. Risk-neutral measure: Girsanovs Theorem for a single Brownian motion, Recognizing Brownian motion, Risky asset price process with stochastic mean rate of return and volatility, Risk-free interest rate, Discount process with adapted interest rate process, Risk-neutral measure, Discounted risky asset price process under risk-neutral measure, Portfolio value process, Discounted portfolio value process under Risk-Neutral Measure. Pricing under the risk-neutral measure. Deriving the Black-Scholes-Merton formula.