Real Analysis I: MATH 8143

August 2023 Exam

Text: Real Analysis, 4th edition, Prentice Hall, H.L. Royden and P.M. Fitzpatrick

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Real Numbers
- The Field, Positivity, and Completeness Axioms (1.1)
- The Natural and Rational Numbers (1.2)
- Countable and Uncountable Sets (1.3)
- Open, Closed, and Borel Sets (1.4)
- Sequences (1.5)
- Continuous Real-Valued Functions (1.6)

Lebesgue Measure
- Lebesgue Outer Measure (2.2)
- Lebesgue Measurable Sets (2.3)
- Outer and Inner Approximation (2.4)
- Countable Additivity, Continuity, and the Borel-Cantelli Lemma (2.5)
- Nonmeasurable Sets (2.6)
- The Cantor Set and the Cantor-Lebesgue Function (2.7)

Lebesuge Measurable Functions
- Sums, Products, and Compositions (3.1)
- Sequential Pointwise Limits and Simple Approximation (3.2)
- Egoroff's Theorem and Lusin's Theorem (3.3)

Lebesgue Integration
- The Lebesgue Integral for bounded functions on sets of finite measure (4.2)
- The Lebesgue Integral for nonnegative functions (4.3)
- The General Lebesgue Integral (4.4)
- Countable Additivity and Continuity of Integration (4.5)
- Uniform Integrability and the Vitalli Convergence Theorem (4.6)

Further topics in Lebesgue Integration
- Uniform Integrability and Tightness (5.1)
- Convergence in Measure (5.2)

Differentiation and Integration
- Continuity of Monotone Functions (6.1)
- Differentiability of Monotone Functions (6.2)
- Functions of Bounded Variation (6.3)
- Absolutely Continuous Functions (6.4)
- Integrating Derivatives and Differentiating Indefinite Integrals (6.5)