MATH 1241 FINAL EXAM SPRING 2011

PART I

Name_____ Instructor:_____
Student ID # _____ Section/Time_____

This exam is divided into three parts. **Calculators are not allowed on Part I.** You have three hours for the entire test, but you have only one hour to finish Part I. You must turn in the answer sheet for Part I at 9:00 am. You may start working on the other two parts of the exam whenever you are done with Part I, but you cannot use your calculator until ALL of the Part I answer sheets are collected. After these answer sheets are collected, your instructor will announce that calculators are allowed on Parts II and III.

These pages contain Part I which consists of 15 multiple choice questions. These questions must be answered without the use of a calculator.

- You must use a pencil with a soft black lead (#2 or HB) to enter your answers on the answer sheet

- For each question choose the response that best fits the question

- If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.

- There is no penalty for guessing

- If you mark more than one answer to a question, the question will be scored as incorrect.

- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.

- **Make sure that your name appears on the answer sheet and that you fill in the circles corresponding to your name.**

At approximately 9 am, you MUST hand in the answer sheet for Part I. At the end of the exam, you MUST hand in all remaining test materials including test booklets, answer sheet for Part II, and scratch paper.

1. Let $h(x) = (x^2 + 1)^3$. Evaluate $h'(1)$.
(a) 6
(b) 12
(c) 18
(d) 24
(e) 30

2. Let $f(x) = sin(2x)$. Find $f'(x)$.
(a) $cos(2)$
(b) $-cos(2)$
(c) $cos(2x)$
(d) $-cos(2x)$
(e) $2\,cos(2x)$

3. Let $f(x) = 3x^2 ln(3x)$. Evaluate $f'(1)$
(a) $1 + 2 \cdot ln(3)$
(b) $2 + ln(3)$
(c) $3 + 6 \cdot ln(3)$
(d) $6 \cdot ln(3)$
(e) 1

4. Let $f(x) = \frac{x}{x^2-x+1}$. Then the derivative $f'(x)$ equals
(a) $\frac{1}{2x-1}$
(b) $\frac{x^2}{x^2-x+1}$
(c) $\frac{1-x^2}{(x^2-x+1)^2}$
(d) $\frac{1-x}{(x^2-x+1)^2}$
(e) $\frac{1+2x+3x^2}{(x^2-x+1)^2}$

5. Let $f(x) = \sqrt{x^2 - 2x}$. Then the derivative $f'(x)$ equals
(a) $\sqrt{2x-2}$
(b) $\frac{1}{2}\sqrt{x^2-2x}$
(c) $\frac{1}{2\sqrt{x^2-2x}}$
(d) $\frac{x-1}{\sqrt{x^2-2x}}$
(e) $\frac{1}{(2x-2)\sqrt{x^2-2x}}$

11. Find the general antiderivative of the function $f(x) = 2e^x + x^2 - 1$
(a) $2e^x + x^2 - x + C$
(b) $2e^x + x^3 - 1 + C$
(c) $2e^x + \frac{1}{3}x^3 - x + C$
(d) $e^{2x} + \frac{1}{3}x^3 - x + C$
(e) $2e^{2x} + \frac{1}{3}x^3 - x + C$

12. Find the general antiderivative of the function $f(x) = \frac{3}{x} + 2sin(x) - 3cos(x)$
(a) $3 + 2\,cos(x) - 3sin(x) + C$
(b) $3 - 2\,cos(x) - 3sin(x) + C$
(c) $ln|x| - 2cos(x) - 3sin(x) + C$
(d) $3\,ln|x| + 2cos(x) + 3sin(x) + C$
(e) $3\,ln|x| - 2cos(x) - 3sin(x) + C$

13. If the graph of $f(x) = kx^3 - 12x^2 + 5x + 7$ has an inflection point at $x = 2$, then k is equal to
(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

14. Use L'Hospital's rule to evaluate $\lim_{x \to 0} \frac{e^{3x} - 1}{x}$.
(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

15. Let $x^2y^3 - x + 2y = 2$. Find $\frac{dy}{dx}$ at the point $(1, 1)$.
(a) -1
(b) $-\frac{1}{5}$
(c) 0
(d) $\frac{1}{10}$
(e) $\frac{2}{5}$

MATH 1241 FINAL EXAM SPRING 2011

PART III

Name_____ Instructor:_____
Student ID # _____ Section/Time_____

These pages contain Part III which consists of 5 free response questions.

Please show all your work in this test booklet. Loose paper will not be graded.

•If you are basing your answer on a graph on your calculator, sketch this graph in the answer booklet. Be sure to label your window by putting a scale on the axes.

•Make sure that your name appears on each page.

At the end of the exam, you MUST hand in all remaining test materials including test booklets, answer sheet, and scratch paper.

PROBLEM	1	2	3	4	5
GRADE					

FREE RESPONSE SCORE:_____

1. In this problem we will find the x- and y-coordinates of the point on the line $y = 3x + 1$ that is closest to the origin.

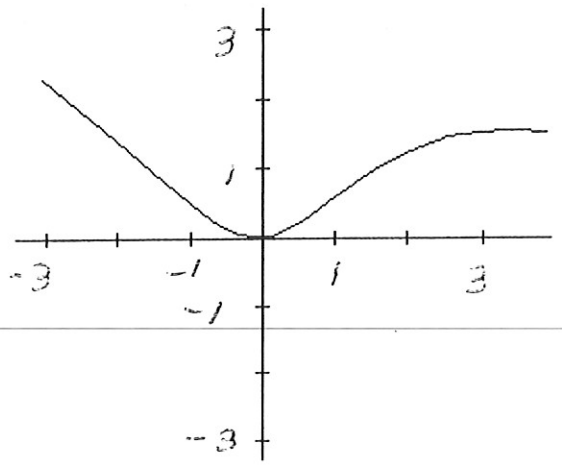
(a) The distance between two points (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Use this distance formula to express the distance between the points (x, y) and $(0, 0)$ as a function of x and y.

(b) Suppose that the point (x, y) is on the line $y = 3x + 1$. Use this fact and the result of part (a) to express the distance between (x, y) and $(0, 0)$ as a function of x alone.

(c) Use calculus to determine the value of x that minimizes the distance between a point (x, y) on the line $y = 3x + 1$ and the point $(0, 0)$. Show your work.

(d) If the point $(x, y)$is on the line $y = 3x + 1$, what is the y-value that goes with x in part (c)?

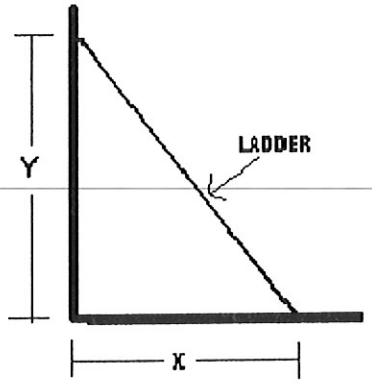
2. Suppose that the graph of $y = f'(x)$ is shown below. (Note: This is the graph of the derivative of f.)



(a) Given that $f(0) = 0$, sketch a graph of $y = f(x)$

(b) Sketch a graph of $y = f''(x)$. (Recall that f'' is the derivative of f')

3. A 25 foot ladder rests against a vertical wall. Let y be the distance between the top of the ladder and the ground and let x be the distance from the bottom of the ladder to the wall. If the bottom of the ladder slides away from the wall at a constant rate of 3 ft per second, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 16 feet from the wall?



(a) The triangle above is a right triangle. Express the length of the ladder in terms of x and y.

(b) Differentiate the expression from part (a) with respect to time t, to get an equation involving $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

(c) What is the value of $\frac{dx}{dt}$? (If you don't know, please re-read the problem.)

(d) What is the value of y when $x = 16$? (Part (a) might be helpful.)

(e) Use the answers to (c) and (d) to find $\frac{dy}{dt}$ when $x = 16$.

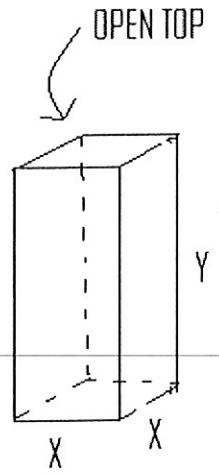
4. (a) Let $f(x) = 3x^2 + 2x$. Use the definition of derivative $f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ to evaluate $f'(2)$. You must show your work. (No credit will be given for answers based on the fast rules.)

(b) The expression $\lim_{h \to 0} \frac{e^{3(1+h)}-e^3}{h}$ represents $f'(a)$ for some function f and some point a.

(1) Find a formula for a function $f(x)$ such that $f'(a) = \lim_{h \to 0} \frac{e^{3(1+h)}-e^3}{h}$.

(2) What is the number a that makes $f'(a) = \lim_{h \to 0} \frac{e^{3(1+h)}-e^3}{h}$?

5. A box with a square base and an open top must have a volume of 5000 cubic inches. We wish to find the dimensions of the box that minimize the amount of material used. (This is the same as minimizing the surface area of the box.)



(a) Write an equation that relates the 5000 cubic inch volume to the variables X and Y in the diagram above.

(b) Write an equation that expresses the surface area of the box in terms of the variables X and Y in the diagram. Remember that the box is open (i.e. it has no top).

(c) Use the answer to (a) to eliminate one of the variables in part (b). Then determine the dimensions of the box that minimize the surface area.

MATH 1241 FINAL EXAM SPRING 2011

MATH 1241 FINAL EXAM SPRING 2011

PART II

Name_____ Instructor:_____
Student ID # _____ Section/Time_____

These pages contain Part II which consists of 15 multiple choice questions. After the answer sheets for Part I have all been collected, and your intstructor announces that calculators are OK, you are allowed to use a calculator on this part of the exam.

- You must use a pencil with a soft black lead (#2 or HB) to enter your answers on the answer sheet

- For each questions choose the response that best fits the question

- If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.

- There is no penalty for guessing

- If you mark more than one answer to a question, the question will be scored as incorrect.

- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.

- **Make sure that your name appears on the answer sheet for Part II and that you fill in the circles corresponding to your name.**

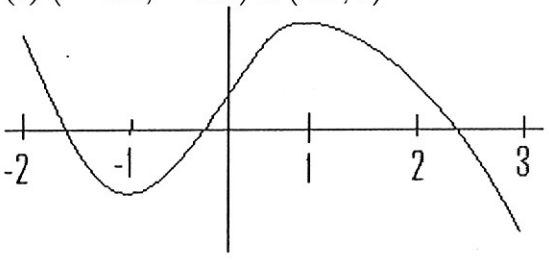
At the end of the exam, you MUST hand in all remaining test materials including test booklets, answer sheet for Part II, and scratch paper.

1. Find the x-coordinate(s) of the point(s) on the curve $y = 4x^3 - 15x^2 - 18x$ where the tangent line is horizontal.
(a) $x = 1,\ x = 2$
(b) $x = -1,\ x = \frac{1}{2}$
(c) $x = -\frac{1}{2},\ x = 3$
(d) $x = -1,\ x = \frac{3}{2}$
(e) $x = 0$ only

2. If the function $f(x) = \begin{cases} x^2, & x \leq 1 \\ 2x + k, & x > 1 \end{cases}$ is continuous at $x = 1$, then k is
(a) -2
(b) -1
(c) 0
(d) 1
(e) 2

3. Let $f(x) = x^4 - 6x^3 - 60x^2$. On which interval is f concave down?
(a) $(-3,\ 2)$
(b) $(-3, 5)$
(c) $(-2, 4)$
(d) $(-2, 5)$
(e) $(-2, 6)$

4. The graph of $y = f'(x)$ is shown. Over which of the following intervals is f concave up? **(Note that you were given the graph of the derivative f', but are being asked about f.)**
(a) $(-2, 0)$
(b) $(1, 3)$
(c) $(-1, 1)$
(d) $(-2, -1.5) \cup (-0.3, 2.4)$
(e) $(-1.5,\ -0.3) \cup (2.4, 3)$



8. Use the information in the table below to evaluate $(g \circ f)'(1)$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	-3	3	-2
2	3	7	1	4
3	1	11	2	5

(a) -12
(b) -6
(c) 0
(d) 7
(e) 28

9. The linear approximation $L(x)$ of the function $f(x) = \sqrt{9+18x}$ at the point $x = 0$ is
(a) $L(x) = 3 + 3x$
(b) $L(x) = 9 + 3x$
(c) $L(x) = 3 + \sqrt{18}\,x$
(d) $L(x) = 9 + \frac{1}{3}x$
(e) $L(x) = 18 + \frac{1}{3}x$

10. Evaluate $\lim_{x \to 0^-} \frac{x}{|x|}$
(a) -1
(b) 0
(c) $\frac{1}{2}$
(d) 1
(e) The limit does not exist

11. Evaluate $\lim_{x \to 2} \frac{x^2+3x-10}{x-2}$.
(a) 1
(b) 3
(c) 5
(d) 7
(e) The limit does not exist.