STAT 1220
Common Final Exam

FALL 2018
December 7, 2018

PLEASE PRINT THE FOLLOWING INFORMATION:

Name: ______________________ Instructor: ______________________

Student ID #: ______________ Section/Time: ______________

THIS EXAM HAS TWO PARTS.

PART I.
Part I consists of 30 multiple choice questions. Each correct answer is scored 2 points; each incorrect (or blank) answer is scored 0, so there is no penalty for guessing. You may do calculations on the test paper, but your answers must be marked on the OPSCAN sheet with a soft lead pencil (HB or No. 2 lead). Any question with more than one choice marked will be counted as incorrect. If more than one choice seems correct, choose the one that is most complete or most accurate. Make sure that your name and ID number are written and correctly bubbled on the OPSCAN sheet.

PART II.
Part II consists of two free response questions, with values as indicated. You must show all work in the space provided or elsewhere on the exam paper in a place that you clearly indicate. Work on loose sheets will not be graded.

FOR DEPARTMENT USE ONLY:

Part II.

Question	1	2
Score		

Part I	Part II	TOTAL

Cumulative Binomial Probability $P(X \leq x)$

n	x	0.05	0.10	0.25	$0.3\bar{3}$	0.50	$0.6\bar{6}$	0.75	0.90	0.95
5	0	0.7738	0.5905	0.2373	0.1317	0.0313	0.0041	0.0010	0.0000	0.0000
	1	0.9774	0.9185	0.6328	0.4609	0.1875	0.0453	0.0156	0.0005	0.0000
	2	0.9988	0.9914	0.8965	0.7901	0.5000	0.2099	0.1035	0.0086	0.0012
	3	1.0000	0.9995	0.9844	0.9547	0.8125	0.5391	0.3672	0.0815	0.0226
	4	1.0000	1.0000	0.9990	0.9959	0.9688	0.8683	0.7627	0.4095	0.2262
20	0	0.3585	0.1216	0.0032	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.7358	0.3917	0.0243	0.0033	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.9245	0.6769	0.0913	0.0176	0.0002	0.0000	0.0000	0.0000	0.0000
	3	0.9841	0.8670	0.2252	0.0604	0.0013	0.0000	0.0000	0.0000	0.0000
	4	0.9974	0.9568	0.4148	0.1515	0.0059	0.0000	0.0000	0.0000	0.0000
	5	0.9997	0.9887	0.6172	0.2972	0.0207	0.0002	0.0000	0.0000	0.0000
	6	1.0000	0.9976	0.7858	0.4793	0.0577	0.0009	0.0000	0.0000	0.0000
	7	1.0000	0.9996	0.8982	0.6615	0.1316	0.0037	0.0002	0.0000	0.0000
	8	1.0000	0.9999	0.9591	0.8095	0.2517	0.0130	0.0009	0.0000	0.0000
	9	1.0000	1.0000	0.9861	0.9081	0.4119	0.0376	0.0039	0.0000	0.0000
	10	1.0000	1.0000	0.9961	0.9624	0.5881	0.0919	0.0139	0.0000	0.0000
	11	1.0000	1.0000	0.9991	0.9870	0.7483	0.1905	0.0409	0.0001	0.0000
	12	1.0000	1.0000	0.9998	0.9963	0.8684	0.3385	0.1018	0.0004	0.0000
	13	1.0000	1.0000	1.0000	0.9991	0.9423	0.5207	0.2142	0.0024	0.0000
	14	1.0000	1.0000	1.0000	0.9998	0.9793	0.7028	0.3828	0.0113	0.0003
	15	1.0000	1.0000	1.0000	1.0000	0.9941	0.8485	0.5852	0.0432	0.0026
	16	1.0000	1.0000	1.0000	1.0000	0.9987	0.9396	0.7748	0.1330	0.0159
	17	1.0000	1.0000	1.0000	1.0000	0.9998	0.9824	0.9087	0.3231	0.0755
	18	1.0000	1.0000	1.0000	1.0000	1.0000	0.9967	0.9757	0.6083	0.2642
	19	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9968	0.8784	0.6415

Part I

Problems 1 and 2 pertain to the following sample data:

4, 0, −3, −1, 5, 2, −2, 1, 3

1. The sample mean of this data set is about

(a) 2.7 (b) 2.3 (c) 2.6 (d) 1.0 (e) 1.1

2. The sample standard deviation of this data set is about

(a) 1.1 (b) 2.7 (c) 1.2 (d) 2.6 (e) 3.3

Problems 3–5 pertain to the data set of 108 measurements represented by the following data frequency table:

x	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
f	2	4	5	8	8	8	9	10	11	10	10	9	5	3	3	2	1

3. The sample median is about

(a) 30.5 (b) 55 (c) 29 (d) 29.5 (e) 30

4. The sample range is about

(a) 108 (b) 60 (c) 16 (d) 38 (e) 55

5. The percentile rank of the measurement 26 is about

(a) 18 (b) 29 (c) 25 (d) 7 (e) 32

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6. The distribution of the amount payable on the December energy bills in a particular residential area is roughly bell-shaped with mean \$270 and standard deviation \$35 years. The proportion of households for which the bill is at least \$200 is about

(a) 0.970 (b) 0.950 (c) 0.680 (d) 0.975 (e) 0.500

7. A medical researcher wishes to estimate the proportion of all adult women who carry a mutation of the BRCA1 gene, which is linked to incidence of breast cancer. In a sample of 12,386 women, 21 carry the mutation. The *population* of interest to the researcher is:

   (a) All adult women except the 12,386 in the sample.
   (b) All the 12,386 women in the sample except those who carry the mutated gene
   (c) All the 12,386 women in the sample.
   (d) All adult women.
   (e) The 21 women in the sample who carry the mutated gene.

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8. The *standard deviation* of a numerical data set measures the ______ of the data.

   (a) difference between the largest and smallest numbers
   (b) variability
   (c) average
   (d) most frequent value
   (e) size

Problems 9 and 10 are based on the following display of the results of a survey done to investigate the relationship between education level and smoking habits.

		Smoking habit	
		Smokes	Does not smoke
Education	Did not finish High School	92	156
	High School diploma	84	298
	College degree	33	211

9. The probability that a randomly selected individual smokes is about:

   (a) 0.163 (b) 0.239 (c) 0.018 (d) 0.211 (e) 0.314

10. The probability that a randomly selected individual smokes, given that he did not finish high school, is about:

   (a) 0.371 (b) 0.284 (c) 0.235 (d) 0.213 (e) 0.404

Problems 11 and 12 are based on the following information: the probability that a randomly selected graduate from a business school has a concentration in accounting is 0.29, the probability that he has a concentration in finance is 0.18, and that he has concentrations in both accounting and finance is 0.08.

11. The probability that a randomly selected business school graduate has a concentration in either accounting or finance (or both) is about:

    (a) 0.39 (b) 0.55 (c) 0.47 (d) 0.11 (e) impossible to tell (insufficient information)

12. The events A: *a randomly selected business school graduate has a concentration in accounting* and F: *a randomly selected business school graduate has a concentration in finance* are:

    (a) Independent because $P(A) \cdot P(F) = P(A \cap F)$
    (b) Independent because $P(A) \cdot P(F) \neq P(A \cap F)$
    (c) Dependent because $P(A) \cdot P(F) = P(A \cap F)$
    (d) Dependent because $P(A) \cdot P(F) \neq P(A \cap F)$
    (e) impossible to tell if independent or not (insufficient information)

Problems 13–15 pertain to the probability distribution of the number X of joint accounts a randomly selected customer of a particular bank has with the bank. (The bank allows any customer a maximum of five joint accounts.)

x	0	1	2	3	4	5
$P(x)$	p	0.54	0.12	0.09	0.04	0.01

13. The missing entry p is about:

    (a) 0.31 (b) 0.05 (c) 0.13 (d) 0.20 (e) 0.11

14. The probability that the number of joint accounts held by a randomly selected customer is more that one is about:

    (a) 0.12 (b) 0.70 (c) 0.26 (d) 0.80 (e) 0.54

15. The average number of joint accounts held per customer is about:

    (a) 1.88 (b) 2.50 (c) 1.00 (d) 1.26 (e) 1.47

16. A salesman will make five cold calls tomorrow. If the probability that a randomly selected cold call results in a sale is 0.10, then the probability that he will make at least one sale from these calls tomorrow is about:

(a) 0.410 (b) 0.328 (c) 0.100 (d) 0.591 (e) 0.238

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17. An automated assembly line produces 43,500 parts per duty cycle. The probability that a randomly selected part will be defective is 0.0023. The average number of defective parts per duty cycle is about:

(a) 217 (b) 100 (c) 435 (d) 23 (e) 44

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18. The actual amount of hot water dispensed by a coffee brewer, when it is set to dispense 8.0 ounces, is normally distributed with mean 8.0 ounce and standard deviation 0.2 oz. The probability that a randomly selected nominal 8.0 ounce cup of coffee will be brewed with more than 8.125 ounces is about:

(a) 0.13 (b) 0.63 (c) 0.20 (d) 0.74 (e) 0.26

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19. In the context of the previous problem, Festus drinks ten nominal 8.0 ounce cups of coffee brewed by this machine each week. The probability that the mean amount of coffee in these ten cups in a randomly selected week is more than 8.125 ounces is about:

(a) 0.13 (b) 0.08 (c) 0.02 (d) 0.24 (e) 0.98

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20. Scores on a common final exam are approximately normally distributed with mean 72 and standard deviation 8. The middle 60% of scores will receive a grade of C. The interval centered at and symmetric about the mean that corresponds to a C is about:

(a) $[65.3, 78.7]$ (b) $[71.2, 72.8]$ (c) $[63.1, 80.9]$ (d) $[68.4, 75.6]$ (e) $[69.9, 74.1]$

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21. A selective university advertises that 96% of its bachelor's degree graduates have, on graduation day, a professional job offer or acceptance in a graduate degree program in their major area of study. In a sample of 227 recent graduates this was true of 209 of them. the probability of obtaining a sample proportion as low as or lower than this, if the university's claim is true, is about:

(a) 0.102 (b) 0.131 (c) 0.001 (d) 0.015 (e) 0.084

22. A sample of the annual earnings of 38 MRI technicians had mean 73.4 and standard deviation 5.9 thousand dollars. A 90% confidence interval for the mean annual earnings of all MRI technicians (in thousands of dollars) is about:

(a) $[71.8, 75.0]$ (b) $[71.2, 75.6]$ (c) $[72.2, 74.6]$ (d) $[71.3, 75.5]$ (e) $[70.4, 76.4]$

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23. A study of 105 objects tested for DNA at crime scenes found that 96 were contaminated with "secondary" DNA (DNA of a person known not to have ever been at the crime scene). A 95% confidence interval for the proportion of all objects at crime scenes so contaminated is about:

(a) 0.914 ± 0.045 (b) 0.914 ± 0.054 (c) 0.914 ± 0.061 (d) 0.914 ± 0.070 (e) 0.914 ± 0.031

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24. The number of cubs in each of seven litters of an endangered species of bear were:

2, 5, 2, 2, 3, 1, 2

Assuming that the population of all such sizes of litters is normally distributed, an 90% confidence interval for their mean is about:

(a) 2.4 ± 0.7 (b) 2.4 ± 0.5 (c) 2.4 ± 0.9 (d) 2.4 ± 0.8 (e) 2.4 ± 1.2

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25. An advertising agency wishes to estimate, with 95% confidence and to within \$50, the mean amount small business owners would be willing to pay for a professionally produced web page for their businesses. Assuming a standard deviation of such amounts to be \$250, the estimated minimum sample size needed to meet these criteria is about:

(a) 62 (b) 35 (c) 10 (d) 103 (e) 97

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26. In the test of hypotheses $H_0 : \mu = 0$ versus $H_a : \mu \neq 0$ with $\alpha = 0.005$, when the sample size is 16 and the population is normally distributed but of unknown standard deviation the rejection region will be the interval

(a) $(-\infty, -3.286] \cup [3.286, \infty)$
(b) $(-\infty, -3.252] \cup [3.252, \infty)$
(c) $(-\infty, -2.947] \cup [2.947, \infty)$
(d) $(-\infty, -2.576] \cup [2.576, \infty)$
(e) $(-\infty, -2.807] \cup [2.807, \infty)$

27. In a test of hypotheses $H_0 : \mu = 21$ versus $H_a : \mu < 21$ in a normally distributed population, the rejection region is the interval $(-\infty, -1.771]$, the value of the sample mean computed from a sample of size 14 is $\bar{x} = 19.2$, and the value of the test statistic is $t = -2.297$. The correct decision and justification are:

(a) Reject H_0 because 19.2 is less than 21.
(b) Reject H_0 because 19.2 lies in the rejection region.
(c) Reject H_0 because $-2.297 < -1.771$.
(d) Do not reject H_0 because the sample is small.
(e) Do not reject H_0 because $-2.297 > -1.771$.

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28. In a test of hypotheses $H_0 : \mu_1 - \mu_2 = 110$ vs. $H_a : \mu_1 - \mu_2 < 110$ samples of sizes 143 and 109 produced the test statistic $z = -1.282$. The p-value (the observed significance) of the test is about:

(a) 110 (b) 1.28 (c) 0.10 (d) 0.90 (e) -1.28

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29. An analyst wishes to test whether the proportion p_1 of purchases of brand new merchandise bought online that is returned to the retailer is greater than the proportion p_2 of merchandise bought at brick and mortar stores that is returned. Assuming that the sample sizes are sufficiently large, the setup of the null and alternative hypotheses for this test must be:

(a) $H_0 : p_1 - p_2 = 0$ vs. $H_a : p_1 - p_2 < 0$
(b) $H_0 : p_1 - p_2 = 0$ vs. $H_a : p_1 - p_2 > 0$
(c) $H_0 : \widehat{p}_1 - \widehat{p}_2 = 0$ vs. $H_a : \widehat{p}_1 - \widehat{p}_2 < 0$
(d) $H_0 : \widehat{p}_1 - \widehat{p}_2 = 0$ vs. $H_a : \widehat{p}_1 - \widehat{p}_2 > 0$
(e) $H_0 : \mu_d = 0$ vs. $H_a : \mu_d > 0$

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30. A study investigating the relationship between a country's annual gross domestic product x (in trillions of dollars) and carbon dioxide emissions y (in millions of metric tons) yielded $r = 0.87$, $s_\varepsilon = 141.9$, and the regression equation $\hat{y} = 199.5x + 56.0$. For each additional trillion dollars in gross domestic product, carbon dioxide emissions

(a) increases by about 0.87 million metric tons, on average
(b) increases by about 141.9 million metric tons, on average
(c) increases by about 199.5 million metric tons, on average
(d) increases by about 56.0 million metric tons, on average
(e) changes by an amount that cannot be determined from the information given

Part II

1. In 2016 the mean household credit card debt (among those accounts which were not paid in full each month) was \$11,184. To test whether it is lower now, a random sample of thirty-six households was taken, giving mean credit card debt \$10,972 with standard deviation \$597. The test is performed at the 1% level of significance.

   (a) State the null and alternative hypotheses for the test. [2 points]

   (b) State the formula for the test statistic and compute its value. Justify your answer. [6 points]

   (c) Construct the rejection region and make a decision. [6 points]

   (d) State a conclusion about the current mean household credit card debt, based on the test you performed. [4 points]

   (e) Compute the p-value (the observed significance) of the test. [2 points]

2. In a random sample of sixteen different industries the percentage employment x in STEM (science, technology, engineering, mathematics) and mean annual compensation y (in thousands of dollars) were recorded, with the following results. The scatter diagram showed a linear trend.

$$1 \leqslant x \leqslant 34 \qquad 36.7 \leqslant y \leqslant 79.6 \qquad \sum x = 150.1 \qquad \sum y = 915$$

$$SS_{xx} = 1303.9 \qquad SS_{xy} = 1503.0 \qquad SS_{yy} = 2660.2 \qquad SSE = 927.78 \qquad s_{\varepsilon} = 8.141$$

(a) Find the linear correlation coefficient for percent employment in STEM and annual compensation. [4 points]

(b) Find the equation of the regression line for predicting y from x. [6 points]

(c) Predict the mean annual compensation in industries in which 30 percent of the workforce are in a STEM field. [4 points]

(d) Give a point estimate for the increase in mean annual compensation of workers in any industry for each additional percentage point increase in the percentage of workers in a STEM field. [2 points]

(e) Widen the estimate in part (d) to a 90% confidence interval. [4 points]

STAT 1220 Formula sheet

Descriptive

$$\bar{x} = \frac{\sum x}{n} \qquad s = \sqrt{\frac{\sum x^2 - \frac{1}{n}(\sum x)^2}{n-1}} \qquad z = \frac{x-\bar{x}}{s} \text{ or } z = \frac{x-\mu}{\sigma}$$

Probability

Complements: $P(A^c) = 1 - P(A)$

Additive Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Conditional Probability: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

Independence: $P(A \cap B) = P(A) \cdot P(B)$ if and only if A and B are independent events

Discrete Random Variables

$$\mu = E(X) = \sum xP(x) \qquad \sigma = \sqrt{\sum (x-\mu)^2 P(x)} = \sqrt{\left[\sum x^2 P(x)\right] - \mu^2}$$

Binomial Random Variable

$$P(x) = \frac{n!}{x!(n-x)!} p^x q^{n-x} \qquad \mu = E(X) = np \qquad \sigma = \sqrt{npq}$$

Sampling Distributions

$$\mu_{\bar{X}} = \mu \qquad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} \qquad \mu_{\hat{p}} = p \qquad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

Sample Sizes for Confidence Intervals

$$n = \frac{(z_{\alpha/2})^2 \sigma^2}{E^2} \quad \text{and} \quad n = \frac{(z_{\alpha/2})^2 \hat{p}(1-\hat{p})}{E^2}$$

Inference conditions		Confidence Interval	Test Statistic	df
Inference About a Single Population Mean				
$n \geq 30$	σ known	$\bar{x} \pm z_{\alpha/2}(\frac{\sigma}{\sqrt{n}})$	$Z = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$	-
	σ unknown	$\bar{x} \pm z_{\alpha/2}(\frac{s}{\sqrt{n}})$	$Z = \frac{\bar{X}-\mu_0}{s/\sqrt{n}}$	-
$n < 30$ and normal population	σ known	$\bar{x} \pm z_{\alpha/2}(\frac{\sigma}{\sqrt{n}})$	$Z = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$	-
	σ unknown	$\bar{x} \pm t_{\alpha/2}(\frac{s}{\sqrt{n}})$	$T = \frac{\bar{X}-\mu_0}{s/\sqrt{n}}$	$n-1$
Inference About a Single Population Proportion				
$\widehat{p} \pm 3\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}} \in [0,1]$		$\widehat{p} \pm z_{\alpha/2}\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}$	$Z = \frac{\widehat{p}-p_0}{\sqrt{\frac{p_0(1D\,p_0)}{n}}}$	-
Inference About Two Population Means				
independent samples	$n_1 \geq 30$ and $n_2 \geq 30$	$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$	$Z = \frac{(\bar{x}_1-\bar{x}_2)-D_0}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}$	-
	$n_1 < 30$ or $n_2 < 30$, σ_1 and σ_2 unknown but assumed equal, normal populations	$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{s_p^2(\frac{1}{n_1} + \frac{1}{n_2})}$	$T = \frac{(\bar{x}_1-\bar{x}_2)-D_0}{\sqrt{s_p^2(\frac{1}{n_1}+\frac{1}{n_2})}}$	$n_1 + n_2 - 2$
		$s_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$		
paired samples, normal population of differences		$\bar{d} \pm t_{\alpha/2}\frac{s_d}{\sqrt{n}}$	$T = \frac{\bar{d}-D_0}{s_d/\sqrt{n}}$	$n-1$
Inference About Two Population Proportions				
$\widehat{p}_j \pm 3\sqrt{\frac{\widehat{p}_j(1-\widehat{p}_j)}{n_j}} \in [0,1]$, $j = 1,2$		$(\widehat{p}_1 - \widehat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\widehat{p}_1\widehat{q}_1}{n_1} + \frac{\widehat{p}_2\widehat{q}_2}{n_2}}$	$Z = \frac{(\widehat{p}_1-\widehat{p}_2)-D_0}{\sqrt{\frac{\widehat{p}_1\widehat{q}_1}{n_1}+\frac{\widehat{p}_2\widehat{q}_2}{n_2}}}$	-

Correlation and Regression

$$SS_{xx} = \sum x^2 - \frac{1}{n}\left(\sum x\right)^2 \qquad SS_{xy} = \sum xy - \frac{1}{n}\left(\sum x\right)\left(\sum y\right) \qquad SS_{yy} = \sum y^2 - \frac{1}{n}\left(\sum y\right)^2$$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx} \cdot SS_{yy}}} \qquad \widehat{y} = \widehat{\beta}_1 x + \widehat{\beta}_0 \quad \text{where} \quad \widehat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} \quad \text{and} \quad \widehat{\beta}_0 = \bar{y} - \widehat{\beta}_1\bar{x}$$

$$SSE = SS_{yy} - \widehat{\beta}_1 SS_{xy} \qquad s_\varepsilon = \sqrt{\frac{SSE}{n-2}} \qquad r^2 = \frac{SS_{yy}-SSE}{SS_{yy}} = \frac{SS_{xy}^2}{SS_{xx}SS_{yy}} = \widehat{\beta}_1\frac{SS_{xy}}{SS_{yy}}.$$

$100(1-\alpha)\%$ confidence interval for β_1:

$\widehat{\beta}_1 \pm t_{\alpha/2}\frac{s_\varepsilon}{\sqrt{SS_{xx}}}$ $\quad (df = n-2)$

Test statistic for β_1:

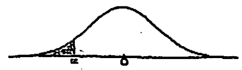
$T = \frac{\widehat{\beta}_1 - B_0}{s_\varepsilon/\sqrt{SS_{xx}}}$ $\quad (df = n-2)$

$100(1-\alpha)\%$ confidence interval for the mean value of y at $x = x_p$:

$\widehat{y}_p \pm t_{\alpha/2}\, s_\varepsilon \sqrt{\frac{1}{n} + \frac{(x_p-\bar{x})^2}{SS_{xx}}}$ $\quad (df = n-2)$

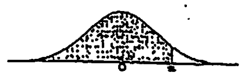
$100(1-\alpha)\%$ prediction interval for an individual new value of y at $x = x_p$:

$\widehat{y}_p \pm t_{\alpha/2}\, s_\varepsilon \sqrt{1 + \frac{1}{n} + \frac{(x_p-\bar{x})^2}{SS_{xx}}}$ $\quad (df = n-2)$



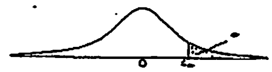
Cumulative Normal Probability $P(Z \leq z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
−3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641



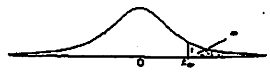
Cumulative Normal Probability $P(Z \leq z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8304	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000



Critical Values of t

df	$t_{0.200}$	$t_{0.100}$	$t_{0.050}$	$t_{0.025}$	$t_{0.010}$	$t_{0.005}$	$t_{0.0025}$	$t_{0.001}$	$t_{0.0005}$
1	1.376	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	1.061	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.978	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.941	1.533	2.132	2.776	3.747	4.604	5.698	7.173	8.610
5	0.920	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.906	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.896	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.889	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.883	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.879	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.876	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.873	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.870	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.868	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.866	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.865	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.863	1.333	1.740	2.110	2.576	2.898	3.222	3.646	3.965
18	0.862	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.861	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.860	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.859	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.858	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.858	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.857	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.856	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.856	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.855	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.855	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.854	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.854	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
31	0.853	1.309	1.696	2.040	2.453	2.744	3.022	3.375	3.633
32	0.853	1.309	1.694	2.037	2.449	2.738	3.015	3.365	3.622
33	0.853	1.308	1.692	2.035	2.445	2.733	3.008	3.356	3.611
34	0.852	1.307	1.691	2.032	2.441	2.728	3.002	3.348	3.601
35	0.852	1.306	1.690	2.030	2.438	2.724	2.996	3.340	3.591
36	0.852	1.306	1.688	2.028	2.434	2.719	2.990	3.333	3.582
37	0.851	1.305	1.687	2.026	2.431	2.715	2.985	3.326	3.574
38	0.851	1.304	1.686	2.024	2.429	2.712	2.980	3.319	3.566
39	0.851	1.304	1.685	2.023	2.426	2.708	2.976	3.313	3.558
40	0.851	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
41	0.851	1.303	1.683	2.020	2.421	2.701	2.967	3.301	3.544
42	0.851	1.302	1.682	2.018	2.418	2.698	2.963	3.296	3.538
43	0.851	1.302	1.681	2.017	2.416	2.695	2.959	3.291	3.532
44	8.850	1.301	1.680	2.015	2.414	2.692	2.956	3.286	3.526
45	0.850	1.301	1.679	2.014	2.412	2.690	2.952	3.281	3.520
46	0.850	1.300	1.679	2.013	2.410	2.687	2.949	3.277	3.515
47	0.849	1.300	1.678	2.012	2.408	2.686	2.946	3.273	3.510
48	0.849	1.299	1.677	2.011	2.407	2.682	2.943	3.269	3.505
49	0.849	1.299	1.677	2.010	2.405	2.680	2.940	3.265	3.500
50	0.849	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496



Critical Values of t

df	$t_{0.200}$	$t_{0.100}$	$t_{0.050}$	$t_{0.025}$	$t_{0.010}$	$t_{0.005}$	$t_{0.0025}$	$t_{0.001}$	$t_{0.0005}$
51	0.849	1.298	1.675	2.008	2.402	2.676	2.934	3.258	3.492
52	0.849	1.298	1.675	2.007	2.400	2.674	2.932	3.255	3.488
53	0.849	1.298	1.674	2.006	2.399	2.672	2.929	3.251	3.484
54	0.848	1.297	1.674	2.005	2.397	2.670	2.927	3.248	3.480
55	0.848	1.297	1.673	2.004	2.396	2.668	2.925	3.245	3.476
56	0.848	1.297	1.673	2.003	2.395	2.667	2.923	3.242	3.473
57	0.848	1.297	1.672	2.002	2.394	2.665	2.920	3.239	3.470
58	0.848	1.296	1.672	2.002	2.392	2.663	2.918	3.237	3.466
59	0.848	1.296	1.671	2.001	2.391	2.662	2.916	3.234	3.463
60	0.848	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
61	0.848	1.296	1.670	2.000	2.389	2.659	2.913	3.229	3.457
62	0.848	1.295	1.670	1.999	2.388	2.657	2.911	3.227	3.454
63	0.847	1.295	1.669	1.998	2.387	2.656	2.909	3.225	3.452
64	0.847	1.295	1.669	1.998	2.386	2.655	2.908	3.223	3.449
65	0.847	1.295	1.669	1.997	2.385	2.654	2.906	3.220	3.447
66	0.847	1.295	1.668	1.997	2.384	2.652	2.904	3.218	3.444
67	0.847	1.294	1.668	1.996	2.383	2.651	2.903	3.216	3.442
68	0.847	1.294	1.668	1.995	2.382	2.650	2.902	3.214	3.439
69	0.847	1.294	1.667	1.995	2.382	2.649	2.900	3.213	3.437
70	0.847	1.294	1.667	1.994	2.381	2.648	2.899	3.211	3.435
71	0.847	1.294	1.667	1.994	2.380	2.647	2.897	3.209	3.433
72	0.847	1.293	1.666	1.993	2.379	2.646	2.896	3.207	3.431
73	0.847	1.293	1.666	1.993	2.379	2.645	2.895	3.206	3.429
74	0.847	1.293	1.666	1.993	2.378	2.644	2.894	3.204	3.427
75	0.846	1.293	1.665	1.992	2.377	2.643	2.892	3.202	3.425
76	0.846	1.293	1.665	1.992	2.376	2.642	2.891	3.201	3.423
77	0.846	1.293	1.665	1.991	2.376	2.641	2.890	3.199	3.421
78	0.846	1.292	1.665	1.991	2.375	2.640	2.889	3.198	3.420
79	0.846	1.292	1.664	1.990	2.374	2.640	2.888	3.197	3.418
80	0.846	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
81	0.846	1.292	1.664	1.990	2.373	2.638	2.886	3.194	3.415
82	0.846	1.292	1.664	1.989	2.373	2.637	2.885	3.193	3.413
83	0.846	1.292	1.663	1.989	2.372	2.636	2.884	3.191	3.412
84	0.846	1.292	1.663	1.989	2.372	2.636	2.883	3.190	3.410
85	0.846	1.292	1.663	1.988	2.371	2.635	2.882	3.189	3.409
86	0.846	1.291	1.663	1.988	2.370	2.634	2.881	3.188	3.407
87	0.846	1.291	1.663	1.988	2.370	2.634	2.880	3.187	3.406
88	0.846	1.291	1.662	1.987	2.369	2.633	2.880	3.185	3.405
89	0.846	1.291	1.662	1.987	2.369	2.632	2.879	3.184	3.403
90	0.846	1.291	1.662	1.987	2.368	2.632	2.878	3.183	3.402
91	0.846	1.291	1.662	1.986	2.368	2.631	2.877	3.182	3.401
92	0.846	1.291	1.662	1.986	2.368	2.630	2.876	3.181	3.399
93	0.846	1.291	1.661	1.986	2.367	2.630	2.876	3.180	3.398
94	0.846	1.291	1.661	1.986	2.367	2.629	2.875	3.179	3.397
95	0.845	1.291	1.661	1.985	2.366	2.629	2.874	3.178	3.396
96	0.845	1.290	1.661	1.985	2.366	2.628	2.873	3.177	3.395
97	0.845	1.290	1.661	1.985	2.365	2.627	2.873	3.176	3.394
98	0.845	1.290	1.661	1.984	2.365	2.627	2.872	3.175	3.393
99	0.845	1.290	1.660	1.984	2.365	2.626	2.871	3.175	3.392
100	0.845	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
∞ [z]	0.842	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291