MATH 1100 FALL 2010
Common Final Exam December 10, 2010

Please print the following information in case your scan sheet is misplaced:

Name: _____ Instructor: _____

Student ID: _____ Section/Time: _____

The exam consists of 40 multiple choice questions, each of equal value. You may do calculations on this question booklet paper but not on the opscan sheet. Mark beside the number of the opscan sheet corresponding to the test question number in pencil only. Mark only one answer; otherwise the answer will be counted as incorrect. You are not penalized for guessing. Please make sure that your name and student ID appear on the opscan sheet in the spaces provided.

Questions begin on page 1 and be sure to check the back of each page for questions.

At the end of the examination you MUST hand in this booklet, your answer sheet and all scratch paper.

You may use the following formulae:

Factoring: $x^3 - a^3 = (x-a)(x^2+xa+a^2)$ $\quad x^3 + a^3 = (x+a)(x^2-xa+a^2)$

Circle: $(x-h)^2 + (y-k)^2 = r^2$ **Lines :** $y - y_0 = m(x-x_0)$; $y = mx + b$

Quadratic formula: $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ **Parabola Vertex:** $\left(\dfrac{-b}{2a}, f\left(\dfrac{-b}{2a}\right)\right), a \neq 0$

Difference quotient: $\dfrac{f(x+h)-f(x)}{h}$ **Average rate of change:** of $f(x)$ on $[a,b]$: $\dfrac{f(b)-f(a)}{b-a}$

Compound Interest: $A = P\left(1+\dfrac{r}{n}\right)^{nt}$ **Continuous Interest:** $A = Pe^{rt}$

Annuity: $A = \dfrac{P\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{\frac{r}{n}}$ **Sinking Fund:** $P = \dfrac{A\left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^{nt}-1}$

Exponential Growth: $A(t) = A_0e^{rt}$, $r > 0$ **Exponential Decay:** $A(t) = A_0e^{-rt}$, $r > 0$

Logarithms:

$\log_b(xy) = \log_b(x) + \log_b(y)$ $\quad \ln(x) = \log_e(x)$

$\log_b\left(\dfrac{x}{y}\right) = \log_b(x) - \log_b(y)$

$\log_b(x^p) = p\log_b(x)$ $\quad \log_b(x) = \dfrac{\ln x}{\ln b} = \dfrac{\log_{10} x}{\log_{10} b}$

$\log_b b^a = a = b^{\log_b a}$ $\quad \ln e^a = a = e^{\ln a}$

1. Which of the following is a factor of $x^4 - 16$?

   (a) $x - 4$ (b) $x + 4$ (c) $(x + 2)^2$ (d) $x^2 + 4$ (e) $x - 16$

2. Simplify the expression $\left(\frac{x^5}{y^3}\right)^2 \left(\frac{y}{x^{-1}}\right)^5$ and express your answer without any negative exponents.

   (a) $\frac{x^{15}}{y}$ (b) $\frac{x^5}{y}$ (c) x^5y^2 (d) $x^{15}y^2$ (e) $\frac{1}{y}$

3. Simplify the radicals and combine like terms in $2\sqrt{50} - \sqrt{162}$.

   (a) $5\sqrt{2}$ (b) $14\sqrt{2}$ (c) $19\sqrt{2}$ (d) $21\sqrt{2}$ (e) $\sqrt{2}$

4. During a hurricane evacuation a family traveled 260 miles. In the first part of their trip, they averaged 50 mph, but then they had to slow down to 10 mph. If the total time of the travel was 6 hours, how many miles did they drive at 10 mph.

   (a) 5 miles (b) 10 miles (c) 15 miles (d) 20 miles (e) 25 miles

5. What describes best the solution set of the following equation?

$$|7x + 2| + 6 = -1.$$

   (a) This is an identity. (b) It has one solution. (c) It has two solutions.
   (d) It has three solutions. (e) It has no solution.

6. Solve the inequality $|3x - 8| > 1$.

   (a) $\left(-\infty, \frac{7}{3}\right) \cup (3, \infty)$ (b) $\left(\frac{7}{3}, 3\right)$ (c) $(3, \infty)$ (d) $\left(-\infty, \frac{7}{3}\right)$ (e) No solution.

7. Use the discriminant to find the number of real solutions of the equation $x^2 - x - 2010 = 0$.

   (a) No solution. (b) One solution. (c) Two solutions. (d) Three solutions.
   (e) Four solutions.

8. Solve the equation $\sqrt{x+4} = 2 + \sqrt{x-4}$. What describes your solution set best?

   (a) There is no solution. (b) There are two positive solutions.
   (c) There are two negative solutions. (d) One solution, it is negative.
   (e) One solution, it is positive.

9. Use the distance formula to find the distance between $(0, -5)$ and $(12, 0)$.

   (a) 10 (b) 11 (c) 12 (d) 13 (e) 14

10. One endpoint of a line segment is $(0, -5)$ the midpoint is $(-2, 7)$. What is the other endpoint?

    (a) $(-4, 9)$ (b) $(-4, 19)$ (c) $(4, 19)$ (d) $(-1, 1)$ (e) $(1, -1)$

11. Find the center and radius of the circle given by the equation $x^2 - 12x + y^2 - 8y + 49 = 0$.

    (a) Center: $(4, 6)$, Radius: 3 (b) Center: $(6, 4)$, Radius: 3 (c) Center: $(6, 4)$, Radius: $\sqrt{3}$
    (d) Center: $(4, 6)$, Radius: $\sqrt{3}$ (e) Center: $(-6, -4)$, Radius: $\sqrt{3}$

12. What is the slope of the line passing through the points $(0, 5)$ and $(4, 0)$?

    (a) The slope is undefined. (b) $\frac{4}{5}$ (c) $-\frac{4}{5}$ (d) $\frac{5}{4}$ (e) $-\frac{5}{4}$

13. Find the equation of the line passing through $(0, 4)$, perpendicular to the line given by $x + 2y = 3$.

    (a) $y = 2x + 4$ (b) $y = 2x - 4$ (c) $y = -\frac{1}{2}x + 4$ (d) $y = -\frac{1}{2}x - 4$ (e) $y = \frac{1}{2}x + 4$

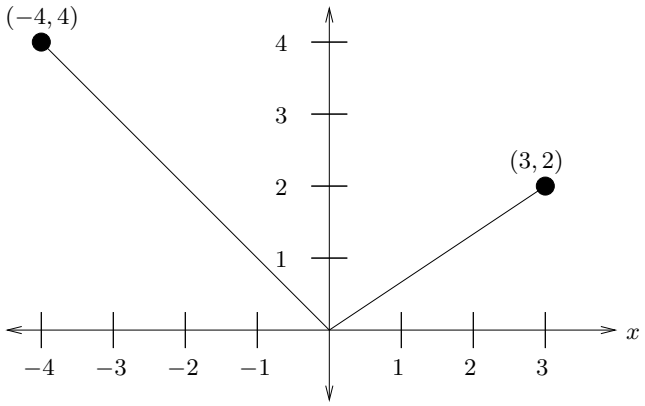
14. The graph of which of the following functions is symmetric to the y axis? In other words, which of the following is an even function?

    (a) $f(x) = 2x + 1$ (b) $f(x) = x^2 - x$ (c) $f(x) = \dfrac{x}{x^3 - x}$ (d) $f(x) = \dfrac{x}{x^2 - 1}$
    (e) $f(x) = \dfrac{x^2 + x}{x^3 - x}$

15. Find the average rate of change of the function $f(x) = x - \sqrt{x}$ on the interval $[1, 4]$.

    (a) -2 (b) $-\frac{2}{3}$ (c) $\frac{5}{3}$ (d) 2 (e) $\frac{2}{3}$

16. Find the formula for the piecewise-defined function shown in the picture.

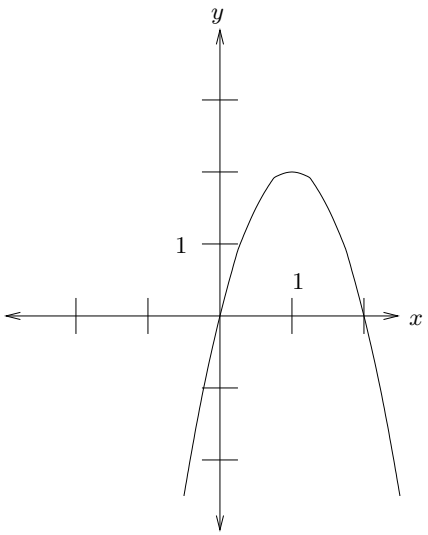


(a) $f(x) = \begin{cases} -x & \text{if } -4 \le x \le 0 \\ \frac{2}{3}x & \text{if } 0 < x \le 3 \end{cases}$ (b) $f(x) = \begin{cases} x & \text{if } -4 \le x \le 0 \\ \frac{2}{3}x & \text{if } 0 < x \le 3 \end{cases}$

(c) $f(x) = \begin{cases} -x & \text{if } -4 \le x \le 0 \\ \frac{3}{2}x & \text{if } 0 < x \le 3 \end{cases}$ (d) $f(x) = \begin{cases} x & \text{if } -4 \le x \le 0 \\ -\frac{3}{2}x & \text{if } 0 < x \le 3 \end{cases}$

(e) $f(x) = \begin{cases} -x & \text{if } -4 \le x \le 0 \\ -\frac{2}{3}x & \text{if } 0 < x \le 3 \end{cases}$

17. Match the correct function to the graph given in the picture.



(a) $f(x) = 2(x-1)^2 + 2$ (b) $f(x) = 2(x-1)^2 - 2$ (c) $f(x) = -2(x-1)^2 + 2$

(d) $f(x) = -2(x+1)^2 + 2$ (e) $f(x) = 2(x+1)^2 + 2$

18. Find the domain of the function $f(x) = \sqrt{4 - x}$.

(a) $(-\infty, \infty)$ (b) $x \neq 4$ (c) $x < 4$ (d) $x \leq 4$ (e) $x \geq 4$

19. Find the composite function $(f \circ g)(x)$ for $f(x) = \sqrt{x + 7}$ and $g(x) = 8x - 11$.

(a) $8\sqrt{x + 7} - 11$ (b) $2\sqrt{2x - 1}$ (c) $8\sqrt{x - 4}$ (d) $2\sqrt{2x + 1}$ (e) $8\sqrt{x} - 4$

20. Find the inverse of the function $f(x) = \dfrac{5}{3}x - \dfrac{4}{3}$.

(a) $f^{-1}(x) = \dfrac{5x + 3}{5}$ (b) $f^{-1}(x) = \dfrac{3x + 4}{5}$ (c) $f^{-1}(x) = \dfrac{4x + 3}{5}$ (d) $f^{-1}(x) = \dfrac{3}{5x - 4}$
(e) The function is not one-to-one.

21. Find the axis of symmetry of the parabola given by $f(x) = x^2 + 4x + 12$.

(a) $x = -2$ (b) $x = 2$ (c) $x = -4$ (d) $x = 4$ (e) $y = 12$

22. A projectile is fired from a boat at an inclination of 45^o to the horizontal, with an initial velocity of 380 feet per second. The height of the projectile above the water is given by $h(x) = -\dfrac{32}{380^2}(x - 2256.25)^2 + 1128.125$ where x is the horizontal distance of the projectile from the boat. What is the highest altitude above water the projectile will reach?

(a) 2256.25 feet (b) 1128.125 feet (c) 2256.25^2 feet (d) $\dfrac{32}{380^2}$ feet (e) 380 feet

23. What are the dimensions of the largest rectangle that can be enclosed by 400 feet of fencing?

(a) 100 ft by 100 feet (b) 200 feet by 200 feet (c) 50 feet by 150 feet
(d) 50 feet by 350 feet (e) 200 feet by 100 feet

24. List each zero of the polynomial $f(x) = (x + 1)^3 \left(x^2 - 4\right)^2$. Also determine whether the function touches or crosses the x-axis at each of the zeros.

(a) $x = -2$, crosses the x-axis; $x = 1$, touches the x-axis; $x = 2$, crosses the x-axis.
(b) $x = -2$, touches the x-axis; $x = 1$, crosses the x-axis; $x = 2$, touches the x-axis.
(c) $x = -2$, touches the x-axis; $x = -1$, crosses the x-axis; $x = 2$, touches the x-axis.
(d) $x = -2$, crosses the x-axis; $x = -1$, touches the x-axis; $x = 2$, crosses the x-axis.
(e) $x = -1$, crosses the x-axis.

25. Use synthetic division to simplify $\dfrac{-5x^3+24x^2-10x-24}{x-4}$.

(a) $-5x^2+4x+6$ (b) $5x-5$ (c) $5x^2+5x-6$ (d) $-\frac{5}{4}x^2+x-\frac{3}{2}$ (e) $-5x^2-4x+6$

26. According to the rational zeros theorem, which numbers would you need to check whether they are rational solutions of the following equation? **Do not solve the equation!**

$$2x^3-x^2+2x-1=0$$

(a) $\dfrac{1}{2}$ (b) $\pm\dfrac{1}{2}$ (c) $\pm\dfrac{1}{2},\pm1$ (d) $\pm1,\pm2$ (e) $\pm\dfrac{1}{2},\pm1,\pm2$

27. Find all vertical asymptotes of the function $f(x)=\dfrac{x-2}{x^2-4}$.

(a) $x=2$ (b) $x=-2$ (c) $x=2$ and $x=-2$ (d) $y=0$
(e) $f(x)$ has no vertical asymptote

28. Find the oblique asymptote of $f(x)=\dfrac{x^2+9x+2}{x+7}$.

(a) $y=-x-2$ (b) $y=-x+2$ (c) $y=x-2$ (d) $y=x+2$ (e) $y=x$

29. Solve the inequality $x(x-1)(3-x)\geq 0$.

(a) $[1,3]$ (b) $(0,1)\cup(3,\infty)$ (c) $[0,1]\cup[3,\infty)$ (d) $(-\infty,0)\cup(1,3)$ (e) $(-\infty,0]\cup[1,3]$

30. Solve the inequality $\dfrac{6x}{5-x}\geq 3x$.

(a) $[0,3]\cup(5,\infty)$ (b) $(5,\infty)$ (c) $(-\infty,3]\cup[5,\infty)$ (d) $(-\infty,0]\cup[3,5)$ (e) $(-\infty,0]\cup[3,5]$

31. Approximate $(2.4)^{2.33}$ using your calculator. Round your answer to three decimal digits.

(a) 7.615 (b) 5.592 (c) 5.765 (d) 7.689 (e) 8.175

32. Since the year 1950, the world population in millions as a function of time is close to the function $A(t)=2600\cdot e^{0.018t}$ where t is the number of years since 1950. Estimate the world population in the year 2018 to the nearest million.

(a) $6,012$ million (b) $8,684$ million (c) $8,842$ million (d) $9,003$ million
(e) $10,914$ million

33. What will be your account balance in 6 years, if you invest $1,000$ dollars at the rate of 3%, compounded quarterly? Round your answer to the nearest dollar.

(a) $1,014$ dollars (b) $1,015$ dollars (c) $1,046$ dollars (d) $1,196$ dollars
(e) $1,197$ dollars

34. The half life of carbon 14 is $5,600$ years. How old is a fossilized leaf that contains 38% of its normal amount of carbon 14? Round your answer to the nearest hundreds of years.

(a) $5,600$ years (b) $7,800$ years (c) $8,300$ years (d) $10,100$ years (e) $1,232,300$ years

35. Solve the equation $4^x = \dfrac{1}{16}$.

(a) -2 (b) 2 (c) $\dfrac{1}{2}$ (d) $-\dfrac{1}{2}$ (e) $-\ln(16)$

36. Assume a and b are positive numbers. Write $\ln\left(\dfrac{9\sqrt{a}}{b}\right)$ as a sum and/or difference of logarithms. Do not use exponents.

(a) $\ln(9) \cdot \dfrac{1}{2}\ln(a)/\ln(b)$ (b) $\ln(9) + \dfrac{1}{2}\ln(a) - \ln(b)$ (c) $\ln(9) - \frac{1}{2}\ln(a) - \ln(b)$
(d) $\ln(9\sqrt{a}) + \ln(b)$ (e) $\dfrac{\ln(9) + \frac{1}{2}\ln(a)}{b}$

37. Use your calculator and the change of base formula to find $\log_3 75$. Round your answer to two decimals.

(a) 0.25 (b) 2.66 (c) 3.14 (d) 3.76 (e) 3.93

38. Solve the equation $\log(x) + \log(x+3) = 1$.

(a) $x = \dfrac{-3 \pm \sqrt{13}}{2}$ (b) $x = \dfrac{-3 + \sqrt{13}}{2}$ (c) $x = -5$ (d) $x = 2$ (e) $x = 2$ or $x = -5$

39. If x and y satisfy the system of equations

$$\begin{aligned} \frac{1}{2}x + y &= 0 \\ x - y &= 3 \end{aligned}$$

Then $x + y$ is equal to

(a) -2 (b) -1 (c) 0 (d) 1 (e) 2

40. Mary is saving up to buy a car by depositing 70 dollars each month into an account that pays 4%, compounded monthly. How long will it take to have enough money in the account to buy a car that costs $11,000$ dollars? Round your answer to the nearest year.

(a) 7 years (b) 8 years (c) 9 years (d) 11 years (e) 13 years