The Braid Index Of Alternating Links

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Abstract

It is well known that the minimum crossing number of an alternating link equals the number of crossings in any reduced alternating link diagram of the link. This remarkable result is an application of the Jones polynomial. In the case of the braid index of an alternating link, Murasugi had conjectured that the number of Seifert circles in a reduced alternating diagram of the link equals the braid index of the link. This conjecture turned out to be false. In this paper we prove the next best thing that one could hope for: we characterize exactly those alternating links for which their braid indices equal to the numbers of Seifert circles in their corresponding reduced alternating link diagrams. More specifically, we prove that if D is a reduced alternating link diagram of an alternating link L, then $b(L)$, the braid index of L, equals the number of Seifert circles in D if and only if $G_S(D)$ contains no edges of weight one. Here $G_S(D)$, called the Seifert graph of D, is an edge weighted simple graph obtained from D by identifying each Seifert circle of D as a vertex of $G_S(D)$ such that two vertices in $G_S(D)$ are connected by an edge if and only if the two corresponding Seifert circles share crossings between them in D and that the weight of the edge is the number of crossings between the two Seifert circles. This result is partly based on the well known MFW inequality, which states that the a-span of the HOMFLY polynomial of L is a lower bound of $2b(L)-2$, as well as a result due to Yamada, which states that the minimum number of Seifert circles over all link diagrams of L equals $b(L)$.

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