Inverse source problems for elliptic and parabolic equations

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Preprint no. 2019-07

Abstract

We solve two inverse problems in this dissertation. The first is the inverse source problem for the Helmholtz equation that governs the wave propagating in anisotropic media. We propose a numerical method to compute a source function from the external measurement of the wave field generated by that source. We derive an equation which is independent of the unknown source. A method to solve it is not yet available, since it's not a standard partial differential equation. By truncating the Fourier series of the wave field with respect to a special basis, we can approximate that equation by a system of elliptic partial differential equations. The solution to this system directly yields the desired source function. We solve it by the quasi-reversibility method.

The second one is to recover the initial condition for parabolic equations from the lateral Cauchy data. We first establish an additional equation for the solution to the parabolic equation, then approximate this equation by an elliptic system, which is solved by the quasi-reversibility method. We present the implementation of our algorithm in details and verify our method by showing some numerical examples.

The convergence of the quasi-reversibility method in both problems are proved using Carlerman estimates.

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