

INVARIANTS OF RATIONAL LINKS REPRESENTED BY  
REDUCED ALTERNATING DIAGRAMS

Yuanan Diao, Claus Ernst and Gábor Heteyi

Preprint no. 2019-13

**Abstract**

A rational link may be represented by any of the (infinitely) many link diagrams corresponding to various continued fraction expansions of the same rational number. The continued fraction expansion of the rational number in which all signs are the same is called a *nonalternating form* and the diagram corresponding to it is a reduced alternating link diagram, which is minimum in terms of the number of crossings in the diagram. Famous formulas exist in the literature for the braid index of a rational link by Murasugi and for its HOMFLY polynomial by Lickorish and Millet, but these rely on a special continued fraction expansion of the rational number in which all partial denominators are even (called *all-even form*). In this paper we present an algorithmic way to transform a continued fraction given in nonalternating form into the all-even form. Using this method we derive formulas for the braid index and the HOMFLY polynomial of a rational link in terms of its reduced alternating form, or equivalently the nonalternating form of the corresponding rational number.

*2010 AMS Subject Classification:* Primary: 57M25; Secondary: 57M27

**Key words and phrases:** continued fractions, knots, links, braid index, alternating, Seifert graph.