A Diagrammatic Approach for Determining the Braid Index of Alternating Links

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Abstract

It is well known that the braid index of a link equals the minimum number of Seifert circles among all link diagrams representing it. For a link with a reduced alternating diagram D, $s(D)$, the number of Seifert circles in D, equals the braid index $\mathbf{b}(D)$ of D if D contains no *lone crossings* (a crossing in D is called a *lone crossing* if it is the only crossing between two Seifert circles in D). If D contains lone crossings, then $\mathbf{b}(D)$ is strictly less than $s(D)$. However in general it is not known how $s(D)$ is related to $\mathbf{b}(D)$. In this paper, we derive explicit formulas for many alternating links based on any minimum projections of these links. As an application of our results, we are able to determine the braid index for any alternating Montesinos link explicitly (which include all rational links and all alternating pretzel links).

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