Braid Index Bounds Ropelength From Below

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Abstract

For an un-oriented link $\mathcal{K}$, let $L(\mathcal{K})$ be the ropelength of $\mathcal{K}$. It is known that in general $L(\mathcal{K})$ is at least of the order $O((Cr(\mathcal{K}))^{3/4})$, and at most of the order $O(Cr(\mathcal{K})\ln^5(Cr(\mathcal{K}))$ where $Cr(\mathcal{K})$ is the minimum crossing number of $\mathcal{K}$. Furthermore, it is known that there exist families of (infinitely many) links with the property $L(\mathcal{K}) = O(Cr(\mathcal{K}))$. A long standing open conjecture states that if $\mathcal{K}$ is alternating, then $L(\mathcal{K})$ is at least of the order $O(Cr(\mathcal{K}))$. In this paper, we show that the braid index of a link also gives a lower bound of its ropelength. More specifically, let $\mathbf{b}(\mathcal{K})$ be the largest braid index among all braid indices corresponding to all possible orientation assignments of the components of $\mathcal{K}$ (called the *maximum braid index* of $\mathcal{K}$), we show that there exists a constant $a > 0$ such that $L(\mathcal{K}) \geq a\mathbf{b}(\mathcal{K})$ for any $\mathcal{K}$. Consequently, $L(\mathcal{K}) \geq O(Cr(\mathcal{K}))$ for any link $\mathcal{K}$ whose absolute braid index is proportional to its crossing number. In the case of alternating links, the absolute braid indices for many of them are proportional to their crossing numbers hence the above conjecture holds for these alternating links.

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