Diffusion processes on solvable groups of upper triangular 3×3 matrices. Applications in Asian and basket options

Julia S. Simonsen

Abstract

One of the questions in algebraic groups is about the asymptotic behavior of the probability of return of a random walk, which closely related on the growth rate of a group. Upper-triangular matrices form a group. Solvable groups have an exponential growth rate and it was shown that the asymptotic behavior of the probability of return on these groups has a fractional-exponential decal. The results in the paper by Molchanov and others, are different from the previous finding. They showed that in the case of solvable groups of upper-triangular 2x2 matrices the return probability of the Brownian motions has a polynomial decay. In this dissertation, we extended this research to the case of solvable groups of upper-triangular 3x3 matrices. The elements in the 3x3 matrices that define a Brownian motion on these groups contain integrals of geometric Brownian motions. These integrals have an important role in Asian and Asian-Basket options. We proved some properties of these integrals and showed that certain cases of geometric Asian-basket call options with two assets have a higher risk that the same type of put options. Which implies that some trading strategies might benefit from a reevaluation using a new risk assessment of geometric Asian-Basket.

Department of Mathematics, UNC Charlotte, Charlotte, NC 28223