Labeling regions in deformations of graphical arrangements

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Abstract

Combining a variant of the Farkas lemma with the Flow Decomposition Theorem we show that the regions of any deformation of a graphical arrangement may be bijectively labeled with a set of weighted digraphs containing directed cycles of negative weight only. Bounded regions correspond to strongly connected digraphs. The study of the resulting labelings allows us to add the omitted details in Stanley's proof on the injectivity of the Stanley-Pak labeling of the regions of the extended Shi arrangement and to introduce a new labeling of the regions in the a-Catalan arrangement. We also point out that Athanasiadis-Linusson labelings may be used to directly count regions in a class of arrangements properly containing the extended Shi arrangement and the Fuss-Catalan arrangement.