Curvature of random walks and random polygons in confinement

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Abstract

The purpose of this paper is to study the curvature of equilateral random walks and polygons that are confined in a sphere. Curvature is one of several basic geometric properties that can be used to describe random walks and polygons. We show that confinement affects curvature quite strongly, and in the limit case where the confinement diameter equals the edge length the unconfined expected curvature value doubles from $\pi/2$ to π. To study curvature a simple model of an equilateral random walk in spherical confinement in dimensions two and three is introduced. For this simple model we derive explicit integral expressions for the expected value of the total curvature in both dimensions. These expressions are functions that depend only on the radius R of the confinement sphere. We then show that the values obtained by numeric integration of these expressions agrees with numerical average curvature estimates obtained from simulations of random walks. Finally, we compare the confinement effect on curvature of random walks with random polygons.

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