A Random Hierarchical Laplacian

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Abstract

The self-similar Hierarchical Laplacian, essentially proposed by Dyson in his theory of 1-D ferromagnetic phase transitions, has a discrete spectrum with each eigenvalue having infinite multiplicity. As a result, the integrated density of states is piecewise constant and the density of states is a sum of point-masses located on its spectrum. To correct these defects, we present a modification of the Hierarchical Laplacian obtained by allowing its deterministic coefficients to instead vary randomly. In this way, the spectrum remains deterministic but the eigenvalues become random with finite multiplicity and we will obtain a continuous density of states. In the last section, we will examine the eigenvalue statistics near an individual point of the spectrum and show that, locally, the spectrum is approximately a Poisson point process.

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