LIMIT THEOREMS FOR RANDOM EXPONENTIAL SUMS AND THEIR APPLICATIONS TO INSURANCE AND RANDOM ENERGY MODEL

Huseyin Erturk

Abstract

In this dissertation, we are mainly concerned with the sum of random exponentials,

$$S_N(t) = \sum_{i=1}^{N(t)} s^{tX_i}$$

Here, $t, N(t) \to \infty$ in appropriate form and $\{X_i, i \geq 1\}$ are i.i.d. random variables. Our first goal is to find the limiting distributions of $S_N(t)$ for new class of the random variables, $\{X_i, i \geq 1\}$. For some classes, such results were known; normal distribution, Weibull distribution etc.

Secondly, we apply these limit theorems to some insurance models and random energy models in statistical physics. Specifically for the first case, we give the estimate of the ruin probability in terms of the empirical data. For REM, we present the analysis of the free energy for new class of distributions of the random variables, X_i. In some particular cases, we prove the existence of several critical points for free energy. In some other cases, we prove the absence of phase transitions.

The technical tool of this study includes the classical limit theory for the sum of i.i.d. random variables and different asymptotic methods like Euler-Maclaurin formula and Laplace method.

Department of Mathematics, UNC Charlotte, Charlotte, NC 28223