

SIMION'S TYPE  $B$  ASSOCIAHEDRON IS A  
PULLING TRIANGULATION OF THE LEGENDRE  
POLYTOPE

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Preprint no. 2016-08

**Abstract**

We show that Simion's type  $B$  associahedron is combinatorially equivalent to a pulling triangulation of a type  $B$  root polytope called the Legendre polytope. Furthermore, we show that every pulling triangulation of the Legendre polytope yields a flag complex. Our triangulation refines a decomposition of the Legendre polytope given by Cho. We extend Cho's cyclic group action to the triangulation in such a way that it corresponds to rotating centrally symmetric triangulations of a regular  $(2n + 2)$ -gon. Finally, we present a bijection between the faces of the Simion's type  $B$  associahedron and Delannoy paths.

*2010 AMS Subject Classification:* Primary: 52B05, 52B12, 52B15; Secondary: 05A15, 05E45