Subsystems of shifts of finite type over countable amenable groups

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Abstract

This dissertation is primarily concerned with the subsystem problem for subshifts of finite type (SFTs) on countable amenable groups. Firstly, we demonstrate that an SFT with positive entropy exhibits a ubiquity of subsystems. We prove that for any countable amenable group G, if X is a G-SFT with positive topological entropy $h(X) > 0$ and $Y \subset X$ is a subshift such that $h(Y) < h(X)$, then the entropies of the SFTs Z which satisfy $Y \subset Z \subset X$ are dense in the in terval $[h(Y), h(X)]$. Secondly, we present an embedding theorem which provides conditions under which a given subshift may be realized as a subsystem of a given SFT. Let G be a countable amenable group with the comparison property. Let X be a strongly aperiodic subshift over G. Let Y be a strongly irreducible shift of finite type over G which has no global period, meaning that the shift action is faithful on Y. If $h(X) < h(Y)$ and Y contains at least one factor of X, then X embeds into Y. Our proofs rely on recent developments in the theory of tilings and quasi-tilings of amenable groups.

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