Dynamics of the shift action on linear sequence spaces over groups beyond $\mathbb{Z}$

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Abstract

In linear dynamics, bounded linear operators over infinite-dimensional Banach spaces have been shown to be able to exhibit interesting characteristics including topological transitivity, topological mixing, and even chaos in the sense of Devaney. This dissertation will examine weighted sequence spaces together with the shift action as the operator. In the case the shift action is over the semi-group , the above topological properties have been previously characterized by conditions on the weight sequence associated with a given weighted space. This work will present recent results for new characterizations of these properties when the group action over a countable group is instead considered. Additionally, an example choice of the weight sequence in this setting will be presented which yields points which are periodic while having an infinite orbit.

Lastly, new implications for infinite and 0 topological entropy for the weighted space with the shift action over will be given. In particular, when the weight sequence is summable over a subset of with positive upper density then infinite entropy may be achieved. Furthermore, when an arbitrary ratio of the weights is bounded above then 0 entropy is guaranteed.

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