

ESSAYS ON λ -QUANTILE DEPENDENT CONVEX
RISK MEASURES

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Abstract

We define a class of convex measures of risk whose values depend on the random variables only up to the λ -quantiles for some given constant $\lambda \in (0, 1)$. For this class of convex risk measures, the assumption of Fatou property can be strengthened, and the robust representation theorem via convex duality method is provided. These results are specialized to the class of λ -quantile law invariant risk measures. We define the λ -quantile uniform preference (λ -quantile second order stochastic dominance) of two probability distribution measures and the λ -quantile dependent concave distortion and study their properties. The robust representation theorem of the λ -quantile dependent Weighted Value-at-Risk is proven via two different approaches: the λ -quantile uniform preference approach and the approach of maximizing the Choquet integral over the core of a λ -quantile dependent concave distortion. We demonstrate the two approaches in a classical example of Conditional Value-at-Risk and a new example of uniform λ -quantile dependent Weighted Value-at-Risk.