Meshless boundary particle methods for boundary integral equations and meshfree particle methods for plates

Christopher Bard Davis

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Abstract

For approximating the solution of partial differential equations (PDE), meshless methods have been introduced to alleviate the difficulties arising in mesh generation using the conventional Finite Element Method (FEM). Many meshless methods introduced lack the Kronecker delta property making them inefficient in handling essential boundary conditions. Oh et al. developed several meshfree shape functions that have the Kronecker delta property. Boundary Element Methods (BEM) solve a boundary integral equation (BIE) which is equivalent to the PDE, thus reducing the dimensionality of the problem by one and the amount of computation when compared to FEM.

In this dissertation, three meshless collocation based boundary element methods are introduced: meshfree reproducing polynomial boundary particle method (RPBPM), patch-wise RPBPM, and patch-wise reproducing singularity particle method (RSBPM). They are applied to the Laplace equation for convex and non-convex domains in two and three dimensions for problems with and without domain singularities.

Electromagnetic wave propagation through photonic crystals is governed by Maxwell's equations in the frequency domain. Under certain conditions, it can be shown that the wave propagation is also governed by Helmholtz equation. Patch-wise RPBPM is applied to the two dimensional Helmholtz equation and used to model electromagnetic wave propagation though lattices of photonic crystals.

For thin plate problems, using the Kirchoff hypothesis, the three dimensional elasticity equations are reduced to a fourth order PDE for the vertical displacement. Conventional FEM has difficulties in solving this because the basis functions are required to have continuous partial derivatives. Suggestions are to use Hermite based elements which are difficult to implement. Using a partition of unity, some special shape functions are developed for thin plates with simple support or clamped boundary conditions. This meshless method for thin plates is then tested and the results are reported.

Department of Mathematics, UNC Charlotte, Charlotte, NC 28223