Limit theorems for reaction diffusion models

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Abstract

We introduce two different reaction diffusion models: evolution of one-cell populations in the presence of mitosis and continuous contact model.

In the first model we consider the time evolution of the supercritical reaction diffusion-equation on the lattice $\mathbb{Z}^d$ when each particle together with it spatial coordinate has an extra parameter (mass). In the moment of the division the mass of the particle which is growing linearly after the birth is divided in random proportion between two offspring (mitosis). Using the technique of moment equations we study asymptotically the mass-space distribution of the particles. For each site in the bulk of the population mass distribution of the particles is the solution of the special differential-functional equation with linearly transformed argument. We prove several limit theorems for such population and study in detail the statistics of the masses of the particles.

The continuous contact model describes the space and time stationary behavior of the particles. The central result here is the existence of limit distributions for continues time critical homogeneous-in-space branching processes with heavy tails spatial dynamics in dimension $d = 2$. In dimension $d \geq 3$, the same results are true without any special assumptions on the underlying (non-degenerated) stochastic dynamics.

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