Polynomial integrability of the Hamiltonian systems with homogeneous potential of degree −3

Jaume Llibre, Adam Mahdi and Claudia Valls

Abstract

In this paper we study the polynomial integrability of natural Hamiltonian systems with two degrees of freedom having a homogeneous potential of degree k given either by a polynomial, or by an inverse of a polynomial. For $k = -2, -1, \ldots, 3, 4$ their polynomial integrability has been characterized. Here we have two main results. First we characterize the polynomail integrability of those Hamiltonian systems with homogeneous potential of degree -3. Second we extend a relation between the nontrivial eigenvalues of the Hessian of the potential calculated at a Darboux point to a family of Hamiltonian systems with potentials given by an inverse of a homogeneous polynomial. This relation was known for such Hamiltonian system with homogeneous polynomial potentials. Finally we present three open problems related with the polynomial integrability of Hamiltonian systems with a rational potential.