Hurwitzian continued fractions containing a repeated constant and an arithmetic progression

Gábor Hetyei

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Abstract

We prove an explicit formula for infinitely many convergents of Hurwitzian continued fractions that repeat several copies of the same constant and elements of one arithmetic progression, in a quasi-periodic fashion. The proof involves combinatorics and formal Laurent series. Using very little analysis we can express their limits in terms of (modified) Bessel functions and Fibonacci polynomials. The limit formula is a generalization of Lehmer's theorem that implies the continuous fraction expansions of e and $\tan(1)$, and it can also be derived from Lehmer's work using Fibonacci polynomial identities. We completely characterize those implementations of our limit formula for which the parameter of each Bessel function is the half of an odd integer, allowing them to be replaced with elementary functions.

Department of Mathematics, UNC Charlotte, Charlotte, NC 28223